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Abstract: This study proposes a method for measuring the degree of automation. First, we relate a task-based model to neoclassical production function. We demonstrate how the automation degree is involved in the production function. Second, we estimate a CES production function. Specifying capital- and labor-input efficiency functions, we calculate the automation degree using the elasticity estimate. Examining Japanese manufacturing industries from 1994 to 2020, we find that the average automation degree across these industries increased slowly, similar to the share of capital income. We also find a significant impact of ICT capital, robotics, and R&D on the automation degree.

Keywords: Task-based model; CES production function; Automation degree; Capital income share.

JEL Classification: E23; O30, O33.

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1. Introduction

Owing to recent advances in Information and Communication Technology (ICT) and Artificial Intelligence (AI), there has been a rapid increase in automation, which suggests the replacement of labor with machines (see Autor and Dorn, 2013; Cortes et al., 2017; Graetz and Michaels, 2017, 2018; Acemoglu and Restrepo, 2020, 2022). Therefore, many studies have explored the impact of automation on the labor market and economic growth (see, for example, Zeira, 1998; Zuleta, 2008; Acemoglu, 2010; Peretto and Seater, 2013; Acemoglu and Restrepo, 2018; Aghion et al., 2019; Hemous and Olsen, 2022; Debraj and Mookherjee, 2022; Nakamura and Zeira, 2024; Acemoglu et al., 2024). According to these theories, the degree of automation, the ratio of automated tasks to the total number of tasks, plays an important role. However, it takes work to count these tasks empirically. The empirical methodology employs data on industrial robotics exposure as a proxy for automation.[1] However, it cannot consider other machines and tools that increase automation. ICT capital and R&D expenditure also significantly increase automation. Therefore, exploring how the degree of automation increases is worthwhile.

This study introduces a novel method for measuring the degree of automation. First, we consider a theory for measuring the degree of automation. By relating a task-based model to a neoclassical production function, we demonstrate how the output per labor unit, the capital per labor unit, and the degree of automation are related. We derive a neoclassical production function consisting of two parts. The first is the relationship between the output and the automation degree. The second is the relationship between the capital per labor unit and the automation degree. This comprehensive approach ensures we can obtain the automation degree consistent with the output and capital per labor unit. Furthermore, we investigate a scenario in which the neoclassical production function reduces a constant elasticity of substitution (CES) production function.

Second, applying our theory, we measure the degree of automation. Considering theoretical and practical advantages compared to other input efficiency specifications, the functions of capital- and labor-input efficiency are assumed to be Pareto distributions, which are deterministic. Owing to this assumption, the degree of automation is connected to these two functions. Examining panel data of 54 Japanese manufacturing industries from 1994 to 2020, we estimated a CES production function and found that the elasticity of substitution between capital and labor is 1.1. We calculated the automation degree using the elasticity estimate. There was a sig-

[1] Mann and Puttermann (2023) considered the share of automation patents to the total patents.

nificant variance among those industries. The average degree of automation across these industries slowly increased from 0.411 to 0.428 during the sample period. The increase in the degree was similar to the share of capital income of the Japanese economy. Examining a subsample of industries with significant investment, we found a more considerable increase in the average degree from 0.407 to 0.454. In addition, using the whole sample, we found a significant positive impact of ICT capital, robotics stock, and R&D expenditure on the automation degree. These findings provide valuable insights into the effects of automation on the Japanese manufacturing industries.

This study introduces a novel method for exploring the degree of automation, a unique contribution to the literature. By connecting a task-based model to a CES production function, we propose a new theory for measuring the degree of automation. This method, which significantly advances the field, allows us to quickly obtain the automation degree using the elasticity estimate, sparking further discussion about the degree's increase.

The remainder of this paper is organized as follows: Section 2 comprehensively reviews the related literature. Section 3 describes the formation of a neoclassical production function through automation. Section 4 thoroughly examines the assumption of labor- and capital-input efficiency functions. Section 5 estimates the CES production function and calculates the automation degree, providing a discussion of the impact of several variables on the degree of automation. Section 6 concludes the study. The Appendix presents proofs and extensions.

2. Related literature

This study is related to four lines of research. The first line employs task-based models (Zeira, 1998; Nakamura and Nakamura, 2008; Nakamura, 2009; Acemoglu, 2010; Acemoglu and Restrepo, 2018; Aghion et al., 2019; Hemous and Olsen, 2022; Nakamura and Zeira, 2024). These studies explore automation, wages, and economic growth by examining the conditions for automation adoption.

Applying our task-based model, we decompose the relationship between output and capital per labor unit into two relationships: the first is between the output and the degree of automation, and the second is between the automation degree and capital per labor unit. Therefore, we can observe the degree of automation in a CES production function.[2]

[2]Several studies examine the robotics capital in a CES production function (Zuleta, 2008; Peretto and Seater, 2013; Berg et al., 2018; Dinlersoz and Zoltan, 2018; Graetz and Michaels, 2018; Gomes, 2018). These studies implicitly consider increased automation by the accumulation

The second line of research is empirical studies on automation. Many studies have empirically investigated the impact of automation on wages, employment, and economic growth (see, for example, Autor and Dorn, 2013; Cortes et al., 2017; Graetz and Michaels, 2017, 2018; Acemoglu and Restrepo, 2020, 2022; Gregory et al., 2022; Adachi et al., 2022; Kikuchi et al., 2024). Acemoglu et al. (2024) explored task shares in which the task share was the integral of input efficiency, with the integral interval being the range of input use. In their study, examining these task shares, the US wage structure was investigated.[3]

This study measures the degree of automation, a crucial aspect of modern industrial processes. We calculated the degree of automation by estimating the elasticity of substitution between capital and labor in a standard CES production function. We also estimated the impact of ICT capital and robotics stock on the degree of automation.

The third line of research, which delves into the microfoundation of a CES production function, is essential. Assuming statistical distributions, several studies have derived either a Cobb-Douglas or CES production function as the aggregate function. Jones (2005) considers capital and labor stochastic coefficients to follow a Pareto distribution and derive a Cobb-Douglas production function. Growiec (2013) examines the Weibull distribution for stochastic coefficients, which implies a CES production function. These findings are crucial in understanding the production functions.

In our task-based model, these two coefficients are deterministic but not stochastic. These coefficients evolve with automation, and a production function is derived from equilibrium. A CES production function is implied when these coefficients satisfy a specific condition. Under the CES production function, we use Pareto distributions to specify capital- and labor-input efficiency functions.[4]

Finally, the fourth line of research regards estimating a CES production function. Many studies have estimated the elasticity of substitution between capital and labor (see a survey by Klump et al., 2012; Knoblach and Stockl, 2020). Examining the Japanese manufacturing industries, we found the elasticity of substitution between

of robotics capital. Our study explicitly explores the degree of automation.

[3]Assuming the production function with the substitution elasticity between tasks being higher than one, the task shares determined the weights of inputs in the aggregate production function with varying elasticity of substitution between capital and labor.

[4]Using task-based models, Nakamura and Nakamura (2008) and Nakamura (2009, 2010) also considered a microfoundation of a CES production function. We overcome the shortcomings of these studies, that is, the lack of a complete examination of the substitution elasticity, the lack of reasons for the assumption of input efficiency functions, and the lack of empirical analysis.

capital and labor to be greater than one but close to one. We calculated the degree of automation using the elasticity estimate. Our proposed method can improve the potency of the CES estimation with its wide application range.

3. Neoclassical production function in our task-based model

Firms are perfectly competitive in a closed economy. The production of goods is considered through a continuum of tasks normalized to unity. Output is assumed to be equal to the minimum value from the tasks:

$$Y_t = \min\{z_t(i) | i \in [0,1]\}, \tag{1}$$

where Y_t is the output at time t and $z_t(i)$ is the input of task i at time t. We assume that in a task, machines and labor are perfectly substitutable:

$$z_t(i) = \theta(i)x_t(i) + \lambda(i)l_t(i), \tag{2}$$

where $x_t(i)$ and $l_t(i)$ are the capital and labor inputs in task i at time t, respectively. $\theta(i)$ and $\lambda(i)$ represent the capital- and labor-input efficiency of task i, respectively. These two input efficiencies are assumed to be positive. We have two reasons for the assumption (1). The first reason is the idea of a bottleneck in a production process. Even with an increase in the input of a task, the output does not increase due to a bottleneck in the other tasks. The second reason is to consider any substitution elasticity between capital and labor under the lowest bound of the substitution elasticity being zero.

Tasks are placed in an order wherein those in which capital is used relatively more efficiently than labor precede those in which capital is used relatively less efficiently than labor:

$$\frac{d\frac{\lambda(i)}{\theta(i)}}{di} \geq 0. \tag{3}$$

We consider the degree of automation, which refers to the range of machine-use tasks. Firms adopt automation in the i-th task if the cost of machines use is lower than that of labor: $\frac{R_t}{\theta(i)} \leq \frac{w_t}{\lambda(i)}$. Assuming that the automation technology is available, the automation condition is

$$\frac{w_t}{R_t} = \frac{\lambda(a_t)}{\theta(a_t)}, \tag{4}$$

where R_t and w_t are the gross interest rate and wage rate at time t, respectively. The gross interest rate is the sum of the interest rate and the depreciation rate by $R_t = r_t + \delta$. δ is the depreciation rate of capital. a_t is the degree of automation, Namely, $1 - a_t$ is the range of labor-use tasks. When the ratio of the interest rate to the wage rate declines, machines replace labor in some tasks. Therefore, automation increases because it lowers the cost of production.

From equation (1), for any $i \in [0, a_t]$ and $j \in (a_t, 1]$, the input quantities satisfy

$$\theta(i)x_t(i) = \lambda(j)l_t(j).$$

Therefore, the following holds:

$$x_t(i) = \frac{\theta(0)x_t(0)}{\theta(i)} \quad and \quad l_t(j) = \frac{\lambda(1)l_t(1)}{\lambda(j)}.$$

Integrating $x_t(i)$ and $l_t(j)$ in the ranges $[0, a_t]$ and $(a_t, 1]$, respectively, we obtain

$$\Theta(a_t)K_t = \theta(0)x_t(0) \quad and \quad \Lambda(a_t)L_t = \lambda(1)l_t(1), \tag{5}$$

where K_t and L_t are the stock of capital and the amount of labor, respectively.

$$K_t \equiv \int_0^{a_t} x_t(i)di \quad and \quad L_t \equiv \int_{a_t}^1 l_t(j)dj.$$

$\Theta(a_t)$ and $\Lambda(a_t)$ represent capital- and labor input efficiencies in aggregate form, respectively.

$$\Theta(a_t) \equiv \left[\int_0^{a_t} \theta(i)^{-1}di\right]^{-1} \quad and \quad \Lambda(a_t) \equiv \left[\int_{a_t}^1 \lambda(j)^{-1}dj\right]^{-1}. \tag{6}$$

The aggregate input efficiency has the following relationship with the task's input efficiency:

$$\frac{d\Theta(a_t)^{-1}}{da_t} = \theta(a_t)^{-1} \quad and \quad \frac{d\Lambda(a_t)^{-1}}{da_t} = -\lambda(a_t)^{-1}. \tag{7}$$

It holds that $\Theta'(a_t) < 0$ and $\Lambda'(a_t) > 0$. Capital input efficiency decreases, but labor input efficiency increases with automation.

Equation (5) implies

$$\Theta(a_t)K_t = \Lambda(a_t)L_t.$$

Therefore, we obtain the following relationship between the degree of automation and the capital/labor ratio:

$$\Omega(a_t) = k_t, \tag{8}$$

where $\Omega(a_t) \equiv \frac{\Lambda(a_t)}{\Theta(a_t)}$ and $k_t \equiv \frac{K_t}{L_t}$. It holds that $\Omega'(a_t) > 0$ because $\Theta'(a_t) < 0$ and $\Lambda'(a_t) > 0$.

From equations (4) and (7), the condition for automation adoption is rewritten as:

$$\frac{w_t}{R_t} = \left[\frac{\Lambda(a_t)}{\Theta(a_t)}\right]^2 \left[-\frac{\Theta'(a_t)}{\Lambda'(a_t)}\right]. \tag{9}$$

As Appendix B.1 shows, equation (3) is equal to the following condition:

$$\frac{d\ln\frac{\Lambda(a_t)}{\Theta(a_t)}}{d\ln a_t} \geq \frac{1}{2}\frac{d\ln\frac{-\Lambda'(a_t)}{\Theta'(a_t)}}{d\ln a_t}. \tag{10}$$

As shown in the following, this assures the concavity of a production function.

Using equation (5), we consider the following Leontief production function in which the coefficients rely on the degree of automation:

$$Y_t = \min\left\{\Theta(a_t)K_t, \Lambda(a_t)L_t\right\}. \tag{11}$$

Equation (8) implies the degree of automation, which is an increasing function of the capital/labor ratio:

$$a_t = a(k_t), \tag{12}$$

where $a(k_t) \equiv \Omega^{-1}(k_t)$ and $k_t \equiv \frac{K_t}{L_t}$. It holds that $a'(k_t) > 0$, $\lim_{k_t \to 0} a(k_t) = 0$, and $a(k_t) \leq 1$. Consequently, the production function (11) with equation (12) can be represented as follows:

$$Y_t = F(K_t, L_t), \tag{13}$$

where $F(K_t, L_t) \equiv \Theta(a(k_t))K_t = \Lambda(a(k_t))L_t$.

We now examine the properties of the production function (13). A production function is defined as a neoclassical production function if the production function is homogeneous degree one and satisfies

$$\frac{\partial Y_t}{\partial K_t} > 0,\ \frac{\partial Y_t}{\partial L_t} > 0,\ \frac{\partial^2 Y_t}{\partial K_t^2} \leq 0,\ \frac{\partial^2 Y_t}{\partial L_t^2} \leq 0,\ \ \textit{and}\ \ \frac{\partial^2 Y_t}{\partial K_t \partial L_t} \geq 0.$$

Proposition 1: (Production function in our task-based model). *Under the assumption (3), the production function (13) equals a neoclassical production function.* Appendix A.1 provides proof.

The neoclassical production function, which describes the relationship between output per labor unit and capital per labor unit, is decomposed into pairs of output per labor unit, degree of automation, and capital per labor unit (Figure 1). Equation (3), namely, equation (10), ensures concavity of the neoclassical production function.[5] The production function per labor unit is described as follows:

$$y_t = f(k_t),$$

where $f(k_t) = \Lambda(a(k_t))$. Under equation (3), $f(k_t)$ satisfies $f'(k_t) > 0$ and $f''(k_t) \leq 0$. Figure 2 illustrates the formation of a neoclassical production function. For

[5] It also implies the second-order condition of equation (11) (Appendix B.2).

simplicity, k_t is assumed exogenously. When the capital/labor ratio increases from k_0 to k_1 to k_2, the degree of automation increases because of a decline in the ratio of the interest rate to the wage rate. The Leontief production function then slides in the north-easterly direction with an increase in automation, for example, from $\Lambda(a_0)$ to $\Lambda(a_1)$ to $\Lambda(a_2)$. Given the arrangement of the tasks in equation (3), the long-run production function satisfies the properties of a neoclassical production function.[6]

4. CES production function in the task-based model

4.1 Condition for the CES production function

In this section, we explore the scenario in which a neoclassical production function reduces a CES production function.

Proposition 2: (CES production function in our task-based model). *Suppose that*

$$b\Theta(a_t)^{\rho} + (1-b)\Lambda(a_t)^{\rho} = \tau, \tag{14}$$

where we assume that $0 < b < 1$ *and* $\tau > 0$. *Our production function (13) is equal to the CES production function:*

$$Y_t = C\left[bK_t^{-\rho} + (1-b)L_t^{-\rho}\right]^{-1/\rho}, \tag{15}$$

where $C = \tau^{\frac{1}{\rho}}$. *When* $\rho = 0$, *equation (14) reduces*

$$b \ln \Theta(a_t) + (1-b) \ln \Lambda(a_t) = \ln \tau. \tag{16}$$

Under equation (16), the Cobb-Douglas production function is:

$$Y_t = cK_t^b L_t^{1-b},$$

where $c = \tau$.
Appendix A.2 provides proof.

In equations (14) and (16), a more (less) rapid decrease in capital-input efficiency symmetrically corresponds to a more (less) rapid increase in labor-input efficiency. Under equations (14) and (16), the neoclassical production function is confined to a CES production function with $\rho \neq 0$ and a Cobb-Douglas production function with $\rho = 0$, respectively. The elasticity of substitution between capital and labor is $\sigma = \frac{1}{1+\rho}$, where $\sigma \geq 0$. Equations (14) and (16) also imply a restriction between the capital- and labor-input efficiencies of a task, namely between $\theta(a_t)$ and $\lambda(a_t)$.

[6] Automation does not appear in TFP (Nakamura and Nakamura, 2009; Prettner, 2019).

The wage and interest rates can be represented by the degree of automation:

$$w_t = C^{-\rho}(1-b)\Lambda(a_t)^{1+\rho} \;\; and \;\; R_t = C^{-\rho}b\Theta(a_t)^{1+\rho}. \tag{17}$$

Appendix B.3 provides proof. When automation increases, the wage rate increases, but the interest rate decreases.

4.2 Degree of automation

We assume capital- and labor-input efficiency functions to calculate the degree of automation. In Subsections 4.2.1 and 4.2.2, we examine when CES and Cobb-Douglas production functions are implied, respectively.

4.2.1 Degree of automation in CES production function

The degree of automation can be considered as a stock variable. Therefore, it can be considered a cumulative distribution function. We examine a statistical distribution when exploring capital- and labor-input efficiency functions.

We denote the capital- and labor-input efficiency as

$$\Theta_t = \Theta(a_t) \;\; and \;\; \Lambda_t = \Lambda(a_t). \tag{18}$$

Equation (18) is rewritten as follows:

$$\begin{aligned} a_t &= H_c(\Theta_t^{-1}) \;\; and \;\; 1-a_t = G_c(\Lambda_t^{-1}) \quad for\ \rho > 0, \\ a_t &= H_s(\Theta_t) \;\; and \;\; 1-a_t = G_s(\Lambda_t) \qquad for\ -1 \le \rho < 0. \end{aligned} \tag{19}$$

We assume that $0 \le H_j(\cdot) \le 1$ and $0 \le G_j(\cdot) \le 1$ $(j = c, s)$. We consider the difference in the representation between $\rho > 0$ and $\rho < 0$ for the consistency with equation (20) in Assumption 1.[7] We make the following assumption.

Assumption 1: The functions, $H_j(X)$ and $G_j(Z)$ are assumed to be the following Pareto cumulative distributions:

$$H_j(X) = 1 - \left(\frac{h_j}{X}\right)^{\xi} \;\; and \;\; G_j(Z) = 1 - \left(\frac{g_j}{Z}\right)^{\zeta}, \tag{20}$$

where $j = c, s$. We assume that $h_j > 0$, $g_j > 0$, $\xi > 0$, and $\zeta > 0$.

In Theorem 1, we consider the functions of capital- and labor-input efficiency, which satisfy the CES restriction (14) and Assumption 1.

[7] Capital and labor are gross complements and gross substitutes when $\rho > 0$ and $\rho < 0$, respectively (see Acemoglu, 2002; Klump et al., 2012; Knoblach and Stockl, 2020). Therefore, we use these abbreviations.

Theorem 1: (Input efficiency with a Pareto distribution). *Suppose equation (14) and Assumption 1. $H_j(\cdot)$ and $G_j(\cdot)$ are represented as follows:*

$$\begin{array}{lll} H_c(\Theta_t^{-1}) = 1 - \frac{b}{\tau}\left(\frac{1}{\Theta_t^{-1}}\right)^{\rho} \quad and \quad G_c(\Lambda_t^{-1}) = 1 - \frac{1-b}{\tau}\left(\frac{1}{\Lambda_t^{-1}}\right)^{\rho} & for\ \rho > 0, \\ G_s(\Theta_t) = 1 - \frac{b}{\tau}\left(\frac{1}{\Theta_t}\right)^{-\rho} \quad and \quad H_s(\Lambda_t) = 1 - \frac{1-b}{\tau}\left(\frac{1}{\Lambda_t}\right)^{-\rho} & for\ -1 \le \rho < 0. \end{array} \tag{21}$$

Appendix A.3 provides proof.

In equations (19) and (21), the degree of automation is connected with capital- and labor-input efficiency functions by Pareto cumulative distributions. Note that these distributions are deterministic but not stochastic. Figure 3.1 illustrates the Pareto cumulative distributions for $0 < \rho$ ($\sigma < 1$). In this figure, the vertical axis is a_t and the horizontal axis is Θ_t^{-1}. Assuming $b = 0.5$ and $\tau = 1$ as a benchmark, we examine $\rho = 0.5$, 1, and 1.5, respectively, corresponding to the broken, solid, and dotted lines. Three lines are interesting when $\Theta_t^{-1} = 1$. A decrease in ρ, which increases the elasticity of substitution between capital and labor, implies a less rapid increase in automation with an increase in Θ_t^{-1}. In Figure 3.2 which corresponds with $-1 \le \rho < 0$ ($\sigma > 1$), the vertical axis is a_t, and the horizontal axis is Λ_t. Assuming $b = 0.5$ and $\tau = 1$, we examine $\rho = -0.2$, -0.5, and -0.8, corresponding to the broken, solid, and dotted lines. The three lines intersect when $\Lambda_t = 1$. An increase in $-\rho$, which increases the elasticity of substitution between capital and labor, implies a more rapid increase in automation with an increase in Λ_t.

Table 1 summarizes capital- and labor-input efficiency functions implied by equations (19) and (21). We observe symmetric relationships between these two. Using equation (7), we also obtain $\theta(i)$ and $\lambda(i)$, which are consistent with the aggregate form. Note that $C = \tau^{\frac{1}{\rho}}$.

Table 1. Capital- and labor-input efficiency functions

	$\Theta(a_t)$	$\Lambda(a_t)$	$\frac{\Lambda(a_t)}{\Theta(a_t)}$
$\rho > 0$	$C\left(\frac{1-a_t}{b}\right)^{\frac{1}{\rho}}$	$C\left(\frac{a_t}{1-b}\right)^{\frac{1}{\rho}}$	$\left(\frac{a_t/(1-b)}{(1-a_t)/b}\right)^{\frac{1}{\rho}}$
$-1 \le \rho < 0$	$C\left(\frac{a_t}{b}\right)^{\frac{1}{\rho}}$	$\left(\frac{1-a_t}{1-b}\right)^{\frac{1}{\rho}}$	$\left(\frac{a_t/b}{(1-a_t)/(1-b)}\right)^{-\frac{1}{\rho}}$
	$\theta(i)$	$\lambda(i)$	$\frac{\lambda(i)/(1-b)}{\theta(i)/b}$
$\rho > 0$	$Cb\rho\left(\frac{1-i}{b}\right)^{\frac{1+\rho}{\rho}}$	$C(1-b)\rho\left(\frac{i}{1-b}\right)^{\frac{1+\rho}{\rho}}$	$\left(\frac{i/(1-b)}{(1-i)/b}\right)^{\frac{1+\rho}{\rho}}$
$-1 \le \rho < 0$	$Cb(-\rho)\left(\frac{i}{b}\right)^{\frac{1+\rho}{\rho}}$	$C(1-b)(-\rho)\left(\frac{1-i}{1-b}\right)^{\frac{1+\rho}{\rho}}$	$\left(\frac{i/b}{(1-i)/(1-b)}\right)^{-\frac{1+\rho}{\rho}}$

We now explain several significant theoretical and practical advantages of these input efficiency functions over other specifications.
(i) Our research's key advantage lies in its straightforward approach to establishing a relationship between the degree of automation and the share of labor and capital income, as elucidated in Corollary 1. This clear and straightforward relationship provides a confident understanding of how automation impacts the distribution of factor incomes.
(ii) Second, we can assure $0 \leq a_t \leq 1$ for any parameter value, as elucidated in Corollary 2.
(iii) Third, we can obtain the degree of automation only by estimating the CES production function. As the automation degree is involved in the CES production function, estimating the other equations and parameters is unnecessary.[8] Therefore, the input efficiency functions noted in Table 1 stand out as a simple and tractable function among the possible functions of input efficiency.
(iv) Finally, we provide the micro foundation of the input efficiency functions noted in Table 1. As demonstrated in Appendix A.4, the input efficiency functions can also be derived using the equilibrium conditions.

Corollary 1 underscores a straightforward relationship between the degree of automation and the share of labor and capital income.

Corollary 1: (Income shares). *Suppose equation (14) and Assumption 1. The income shares are represented by the degree of automation:*

$$\begin{aligned} &\tfrac{w_t L_t}{Y_t} = a_t \quad && and \quad \tfrac{R_t K_t}{Y_t} = 1 - a_t \quad && for\ \rho > 0, \\ &\tfrac{w_t L_t}{Y_t} = 1 - a_t \quad && and \quad \tfrac{R_t K_t}{Y_t} = a_t \quad && for\ -1 \leq \rho < 0. \end{aligned} \tag{22}$$

Appendix A.5 provides proof.

When $\rho > 0$ ($0 \leq \sigma < 1$), the automation degree equals the labor income share. When $-1 \leq \rho < 0$ ($\sigma > 1$), the automation degree equals the share of capital income.

Corollary 2 examines the decomposition of a CES production function.

Corollary 2: (Decomposition of a CES production function). *Suppose equation (14) and Assumption 1. A CES production function is decomposed as follows:*

$$\begin{aligned} &y_t = C\left(\tfrac{a_t}{1-b}\right)^{\frac{1}{\rho}} \quad && and \quad a_t = \tfrac{(1-b){k_t}^{\rho}}{b+(1-b){k_t}^{\rho}} \quad && for\ \rho > 0, \\ &y_t = C\left(\tfrac{1-a_t}{1-b}\right)^{\frac{1}{\rho}} \quad && and \quad a_t = \tfrac{b{k_t}^{-\rho}}{b{k_t}^{-\rho}+(1-b)} \quad && for\ -1 \leq \rho < 0. \end{aligned} \tag{23}$$

[8] If we examine the other statistical distribution, we may need to estimate more parameters that do not appear in a production function.

Appendix A.6 provides proof.

In equation (23), a CES production function is decomposed between the output per labor unit and the degree of automation and between the degree of automation and the capital per labor unit. The output per labor unit is bounded when $\rho > 0$ ($\sigma < 1$). The output per labor unit can increase infinitely when $-1 \leq \rho < 0$ ($\sigma > 1$). The accumulation of capital per labor unit increases automation because it increases the ratio of the wage rate to the interest rate. Regardless of the sign of ρ, it holds that $a(0) = 0$, $\frac{\partial a_t}{\partial k_t} > 0$, $\frac{\partial^2 a_t}{\partial k_t^2} < 0$, and $\lim_{k_t \to \infty} a(k_t) = 1$. Therefore, the degree of automation can asymptotically converge to one when the capital per labor unit increases unboundedly.

Figure 4 illustrates the relationship between output per labor unit and the automation degree and between the automation degree and capital per labor unit. We examine the difference in the elasticity of substitution between capital and labor. When $\rho > 0$ ($\sigma < 1$), $\Lambda(0) = 0$ and $\lim_{k_t \to \infty} \Lambda(a(k_t)) = (1-b)^{-1/\rho}$. $\Lambda(a_t)$ is a concave function. Capital accumulation increases automation regardless of $\rho > 0$ or $\rho < 0$. The automation degree asymptotically converges to one. When $-1 \leq \rho < 0$ ($\sigma > 1$), it holds that $\Lambda(0) > 0$ and $\lim_{a_t \to 1} \Lambda(a_t) = \infty$, implying a convex function of $\Lambda(a_t)$. Full automation can be asymptotically realized when the capital per labor unit increases infinitely. Like $\rho > 0$, the automation degree increases with the capital per labor unit and asymptotically converges to one. As the production function approaches the AK type, the difficulty in automation decreases because of greater substitutability between capital and labor.

4.2.2 Degree of automation in the Cobb-Douglas production function

This subsection considers capital- and labor-input efficiency functions when $\rho = 0$. We prioritize the continuity of the automation degree around $\rho = 0$ while losing some of several advantages obtained under $\rho \neq 0$, which are explained in the previous section.

We assume the capital input efficiency function as follows:

$$\Theta(a_t) = C\left(1 - \rho\frac{a_t - b}{b}\right)^{\frac{1}{\rho}}. \tag{24}$$

The CES restriction (14) implies the labor-input efficiency function:

$$\Lambda(a_t) = C\left(1 + \rho\frac{a_t - b}{1-b}\right)^{\frac{1}{\rho}}. \tag{25}$$

Using (7), we obtain the task's capital- and labor-input efficiency functions: $\theta(i) = Cb\left(1-\rho\frac{i-b}{b}\right)^{\frac{1+\rho}{\rho}}$ and $\lambda(i) = C(1-b)\left(1+\rho\frac{i-b}{1-b}\right)^{\frac{1+\rho}{\rho}}$.

Corollary 3: (Decomposition of a Cobb-Douglas production function). *Suppose equations (14) and (24). When $\rho = 0$, we obtain a Cobb-Douglas production function, which is decomposed as follows:*

$$y_t = c\exp\left(\frac{a_t - b}{1-b}\right) \quad \textit{and} \quad a_t = b(1-b)\ln k_t + b. \tag{26}$$

Appendix A.7 provides proof.

As shown in the proof, we can provide the continuity of the automation degree around $\rho = 0$.[9] Therefore, when ρ takes a value close to zero, we can use equations (26) to measure the degree of automation. We also provide the micro foundation of the capital- and labor-input efficiency functions in equations (24) and (25). These input efficiency functions can also be derived using the equilibrium conditions (see Appendix A.8). However, these two functions cannot imply the straightforward relationship between the automation degree noted in Corollary 1. In addition, when calculating the degree of automation by equation (26), we may need to assume a parameter to assure $0 \leq a_t \leq 1$ (see Appendix A.8).

4.3 Factor-augmenting technical change

We consider a change in input efficiency exogenously, which implies a factor-augmenting technical change. Equation (2) is rewritten as follows:

$$z_t(i) = A_{K,t}\theta(i)x_t(i) + A_{L,t}\lambda(i)l_t(i),$$

where we assume that $A_{K,t} > 0$ and $A_{L,t} > 0$. Equation (6) implies:

$$A_{K,t}\Theta(a_t)K_t = A_{L,t}\Lambda(a_t)L_t.$$

Equation (8) is rewritten as

$$\frac{\Lambda(a_t)}{\Theta(a_t)} = \frac{A_{K,t}}{A_{L,t}}k_t.$$

Consequently, the production function is:

$$Y_t = A_{K,t}\Theta(a_t)K_t = A_{L,t}\Lambda(a_t)L_t.$$

[9] Henningsena and Henningsenb (2012) pointed out convergence problems and unstable results in the CES estimation around $\rho = 0$, and suggested solutions.

Now, we consider a CES production function with factor-augmented technical progress. Regardless of technical progress, the CES production function must satisfy equation (14). We have a CES production function with capital- and labor-augmenting technical progress:

$$Y_t = [b(A_{K,t}k_t)^{-\rho} + (1-b)(A_{L,t}L_t)^{-\rho}]^{-1/\rho}. \tag{27}$$

See Appendix B.4. Note that we normalize $\tau = 1$ in equation (14). When $A_t = A_{K,t} = A_{L,t}$ holds, it implies the CES production function with neutral technical progress. The wage rate and interest rate are

$$w_t = A_{L,t}(1-b)\Lambda(a_t)^{1+\rho} \ \ and \ \ R_t = A_{K,t}b\Theta(a_t)^{1+\rho}.$$

The degree of automation is:

$$a_t = \begin{cases} \frac{(1-b)k_{e,t}^{\rho}}{b+(1-b)k_{e,t}^{\rho}} & for \quad \rho > 0, \\ \frac{bk_{e,t}^{-\rho}}{bk_{e,t}^{-\rho}+(1-b)} & for \quad -1 \leq \rho < 0, \end{cases} \tag{28}$$

where $k_{e,t}$ is the capital per labor unit in efficiency units: $k_{e,t} = \frac{A_{K,t}}{A_{L,t}}k_t$. On the one hand, an increase in capital use efficiency implies capital-augmenting technical progress and increases the automation degree: $\frac{\partial a_t}{\partial A_{K,t}} > 0$. On the contrary, an increase in the efficiency of labor use implies labor-augmenting technical progress and a decrease in the automation degree: $\frac{\partial a_t}{\partial A_{L,t}} < 0$. Neutral technical progress does not affect the degree of automation.

Finally, we consider the Cobb-Douglas production function with technical progress: $Y_t = (A_{K,t}K_t)^b(A_{L,t}L_t)^{1-b}$. The degree of automation is $a_t = \frac{(1-b)k_{e,t}}{b+(1-b)k_{e,t}}$.

5. Degree of automation with estimating a CES production function

5.1 Data description

The data were obtained from the Japan Industrial Productivity (JIP) database 2023. We examined 54 manufacturing industries from 1994 to 2020. The sample size was 1440.[10] We defined several variables to estimate a CES production function. $Y_{i,t}$ is the real value-added output of industry i in period t. $K_{i,t}$ is the capital stock for industry i in period t, $L_{i,t}$ is the person-hour labor for industry i in period t. Regarding $Y_{i,0}$, $K_{i,0}$, and $L_{i,0}$, we use the Divisa index. In the estimation of a CES production function, we employed the FOC conditions. In these FOC conditions, we used the capital and labor cost shares: $\frac{R^n_{i,t}K_{i,t}}{TC^n_{i,t}} = \frac{R^n_{i,t}K_{i,t}/P_{i,t}}{TC^n_{i,t}/P_{i,t}} = \frac{R_{i,t}K_{i,t}}{Y_{i,t}}$ and $\frac{w^n_{i,t}L_{i,t}}{TC^n_{i,t}} =$

[10] We removed 18 data in one of 54 industries because the real added value of output was negative.

$\frac{w^n_{i,t}L_{i,t}/P_{i,t}}{TC^n_{i,t}/P_{i,t}} = \frac{w_{i,t}L_{i,t}}{Y_{i,t}}$. Note that $TC^n_{i,t} = R^n_{i,t}K_{i,t} + w^n_{i,t}L_{i,t}$. $P_{t,i}$ is the price level of output. $R^n_{i,t}$ and $w^n_{i,t}$ are the nominal interest and wage rates, respectively. $R_{i,t}$ and $w_{i,t}$ are the real interest and wage rates, respectively.

In estimating the automation condition, we examined the ICT capital stock, the industrial robotics stock, and R&D expenditure. The ICT capital stock includes information, communication, and software. $va_{i,t}$, $vb_{i,t}$, and $vc_{i,t}$ are the ratio of ICT capital stock to labor, robot stock to labor, and R&D expenditure to labor. These three are also available from the JIP database. The data for industrial robotics stock for industry i in period t are calculated using the Rath method (Moriwaki et al., 2024). We examine the Instrumental Variable (IV) estimation with the instrumental variables, which include the ratio of workers over fifty-five to the total workers and the user cost of ICT capital stock. The data for workers over fifty-five were obtained from the JIP 2023. The user cost of ICT capital stock is calculated by the ratio of ICT capital cost to the total cost of production and the ratio of GDP to ICT capital stock.

5.2 Estimation of a CES production function

We apply the normalization procedure of a CES production function developed by de La Grandville (1989) because we can observe the exact impact of the elasticity of substitution between capital and labor on output. This procedure also alleviates convergence problems and unstable results. We estimate the CES production function with the FOCs. Rewriting the FOCs, we examine the cost shares of capital and labor that help us to have a robust estimation because of the linearity of ρ. The estimation equations are as follows:

$$\ln \frac{Y_{i,t}}{Y_{i,0}} = -\frac{1}{\rho} \ln \left[b_i \left(\exp(g_K(t-t_0)) \frac{K_{i,t}}{K_{i,0}} \right)^{-\rho} + (1-b_i) \left(\exp(g_L(t-t_0)) \frac{L_{i,t}}{L_{i,0}} \right)^{-\rho} \right] + \epsilon_{Yi,t}, \tag{29}$$

$$\ln \frac{R_{i,t}K_{i,t}}{Y_{i,t}} = \ln b_i + \rho \left(\ln \frac{Y_{i,t}}{K_{i,t}} - \ln \frac{Y_{i,0}}{K_{i,0}} \right) - \rho g_K(t-t_0) + \epsilon_{Ri,t}, \tag{30}$$

$$\ln \frac{w_{i,t}L_{i,t}}{Y_{i,t}} = \ln(1-b_i) + \rho \left(\ln \frac{Y_{i,t}}{L_{i,t}} - \ln \frac{Y_{i,0}}{L_{i,0}} \right) - \rho g_L(t-t_0) + \epsilon_{wi,t}. \tag{31}$$

where $i = 1, \cdots, 54$ and $t = 1994, 1995, \cdots, 2020$. The three error terms, $\epsilon_{Yi,t}$, $\epsilon_{Ri,t}$, and $\epsilon_{wi,t}$, are assumed to follow $(\mathbf{0}, \mathbf{\Sigma})$. We assume that $g_K \geq 0$ and $g_L \geq 0$. The production function is the Hicks-natural type if it holds that $g_K = g_L$.

We simultaneously estimated equations (29)–(31) using a Nonlinear Seemingly Unrelated Regression (NSUR) estimation, which was a Feasible Generalized Least Squares (FGLS) estimation. In practice, we sampled geometric averages from the data to calculate $Y_{i,0}$, $K_{i,0}$, and $L_{i,0}$. For t_0, we used the arithmetic average of $t = 1994, 1995, \cdots, 2020$. In practice, b_i was fixed using the arithmetic average of capital share from the data before the estimation.[11] We have two estimations. In the first estimation, equations (29)–(31) are estimated with the difference between g_K and g_L. In the second estimation, we assume that $g_K = g_L$. Under this assumption, technical progress is denoted as g. The estimated result is presented in Table 2.

Table 2. Estimation result of the CES production function

(29) − (31)	ρ	g_K	g_L
	-0.0919	0.0188	0.0055
	(-8.55^{**})	(1.40)	(0.65)
$g_K = g_L$	ρ	g	
	-0.110	0.0111	
	(-11.22^{**})	(2.26^{*})	

Note. The sample size is 1440. Numbers in parentheses show the asymptotic t-values. Cluster-robust standard errors were calculated. $*$ and $**$ represent significance at the 5% and 1% levels, respectively.

When we assumed the difference between g_K and g_L, neither labor- nor capital-augmenting technical progress was significant.[12] The elasticity of substitution is $\hat{\sigma} = (1 + \hat{\rho})^{-1} = 1.10$. The elasticity estimate was significantly greater than one. This implies that in production, capital and labor were imperfectly substitutable. According to the survey of Klump et al. (2007) and Leon-Ledesma et al. (2010), the elasticity estimate differed depending on a sample's data. Several studies found the elasticity estimate to be higher than one while it was close to one (see, for example, Duffy and Papageorgiou, 2000; Hubmer, 2018).[13] The finding of a substitutable relationship between capital and labor is consistent with Japan's declining share of labor incomes. The neutral technical change was significantly positive when we considered Hicks-neutral technical progress with the assumption $g_K = g_L$. The elasticity of substitution is $\hat{\sigma} = (1 + \hat{\rho})^{-1} = 1.12$. Therefore, the elasticity estimate was robust, regardless of the type of technical progress.

[11] Assuming $b_i = b$, we examined the GMM estimation with Kmenta's approximation of a CES production function. The result depended on the instruments.

[12] If we consider the creation of new tasks, the aggregate production function has an endogenous increase in terms of biased technical progress. Our result implies no need to consider new tasks because of no increase in these terms.

[13] In Japan, the union membership rate has been declining. It may imply a large elasticity estimate.

5.3 Degree of automation implied by a CES production function

In Subsection 5.3.1, we delve into the degree of automation with the CES estimation of the entire sample. However, it might be overly simplistic to assume the same elasticity of substitution across all manufacturing industries. Therefore, in Subsection 5.3.2, we narrow our focus to a subsample of industries with substantial investment. By estimating the substitution elasticity of these industries, we aim to highlight increased automation.

5.3.1 Degree of automation using the whole sample

Using the estimation result of the CES production function noted in Table 2, we calculate the degree of automation as follows:

$$a_{i,t} = \frac{b_i \left(\frac{k_{i,t}}{k_{i,0}}\right)^{-\rho}}{b_i \left(\frac{k_{i,t}}{k_{i,0}}\right)^{-\rho} + (1 - b_i)}. \tag{32}$$

This corresponds with equation (28) under $\rho > 0$ and $g_K = g_L$. In calculating the automation degree, we examined the estimate ρ. The descriptive statistics for the degree of automation are presented in Table 3. The sample size is 1440.

Table 3. Degree of automation calculated by equation (32)

mean	*s.e.*	*min*	*max*
0.422	0.172	0.137	0.815

The average degree of automation was 0.422. The difference between the minimum and maximum values was considerably large, with significant variance among manufacturing industries. Therefore, the degree varies among these industries. The average degree of automation across these industries increased from 0.411 to 0.428 during the sample period. Figure 5 illustrates the increase in the average degree by the blue line. The degree slowly increased over time. In Figure 6, assuming a Gaussian kernel function, we illustrate the kernel density of the automation degree in 1994 and 2020. The horizontal and vertical axes are the degree of automation and density, respectively. In these two years, the degree of automation varies among industries. The density has a fat tail with a low degree of automation. The mode of the distribution was a little over 0.3. However, in several industries, the degree of automation exceeds 0.8. Therefore, in those industries, automation has increased much in tasks. From 1994 to 2020, the distribution moves slowly to the right with increased automation. The slow increase in automation was not surprising because, during the sample period, the Japanese economy experienced a prolonged recession, and the real wage rate did not increase.[14]

Finally, we consider the reliability of the degree of automation. According to the annual report of the Ministry of Health, Labor, and Welfare (2023), from 1994 to 2020, the share of capital income increased from 0.373 to 0.428, with an average of 0.41. Therefore, the increase in the average degree of automation across manufacturing industries was similar to the share of capital income. This finding is consistent with Corollary 1, which shows the equality between the automation degree and the share of capital income under $\sigma > 1$.

Our examination of the relationship between the output level and the degree of automation confirms the degree's reliability. Using equation (21), we consider the degree of automation implied by the output level with a normalization:

$$\left(\frac{y_{i,t}}{y_{i,0}}\right) = \exp(g(t-t_0)) \left(\frac{1-a_{i,t}}{1-b_i}\right)^{1/\rho}. \tag{33}$$

Note that C in equation (21) is substituted by neutral technical progress. Rewriting equation (33), the degree of automation is calculated by the observed output level and the estimates of g and ρ obtained from the CES estimation:

$$a_{i,t}|_y = 1 - (1-b_i)\exp[-\rho g(t-t_0)] \left(\frac{y_{i,t}}{y_{i,0}}\right)^{\rho}. \tag{34}$$

Using Figure 7, we explain the difference in the degree of automation between equations (32) and (34). Assuming $\rho < 0$, we illustrate the relationship between output per labor unit and the automation degree and between the automation degree and capital per labor unit. On the one hand, given the value of capital per labor unit, equation (32) represents the degree of automation, which in turn implies the predicted output per labor unit, $\hat{y}_{i,t}$ in the CES estimation. On the other hand, in equation (34), the degree of automation corresponds with the observed output per labor unit. Comparing $a_{i,t}$ with $a_{i,t}|_y$ with the same capital per labor unit, we can see how the automation degree calculated by the capital per labor unit predicts the output level well. Therefore, the precision of the automation degree in our study is measured by the sum of squares of the gap in the automation degree between equations (32) and (34):

$$d = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(a_{i,t}|_y - a_{i,t})^2. \tag{35}$$

[14]In Appendix C, estimating the condition for automation adoption, we also examined the degree of automation.

This precision depends on the CES estimation and the assumption of capital- and labor-input efficiency functions.[15]

Using the entire sample with $NT = 1440$, we found $d = 0.000378$, close to zero. This indicates that the automation degree calculated by the capital per labor unit, as implied by equation (32), explains the output level well, demonstrating the accuracy of the CES estimation and the assumption of the input efficiency function. By choosing the initial and last years of the sample, we check the gap in the automation degree between equations (32) and (34) in detail. In Figures 8.1 and 8.2, which correspond with 1994 and 2020, respectively, assuming a Gaussian kernel function, we illustrate the kernel density of the degree of automation. The blue and red lines show the degree using equations (32) and (34), respectively. In 1994, there was a small gap in density between these two degrees when the degree exceeded 0.4. However, in 2020, the degree predicted the output level more precisely.

5.3.2 Degree of automation for industries having significant investment

In this subsection, we examine a subsample of industries with significant investment. Examining the annual growth rates for capital per labor unit from 1994 to 2020, the average of 54 industries was 2.37%. Regarding industries with substantial investment, we chose 25 sectors, which exceeded the average of the 54 industries.[16] Using this subsample, we simultaneously estimated equations (29)–(31) by a FGLS estimation.

The estimated result is presented in Table 4. The elasticity of substitution is $\hat{\sigma} = (1 + \hat{\rho})^{-1} = 1.22$. The elasticity estimate was larger than that of all industries. Therefore, capital and labor were more substitutable in industries with significant investment. The neutral technical change was significantly positive, similar to the whole sample.

Table 4. CES estimation result of industries having significant investment

$g_K = g_L$	ρ	g
	-0.180	0.0173
	(-15.70^{**})	(2.34^{*})

Note. The sample size is 675. Numbers in parentheses show the asymptotic t-values. Cluster-robust standard errors were calculated. $*$ and $**$ represent significance at the 5% and 1% levels, respectively.

[15] When we examine several functions of input efficiency, equation (35) can be used as the criterion to choose the optimal function of input efficiency function.

[16] These 25 industries included pharmaceutical products, machinery, and electronic equipment.

Using equation (32), we calculated the degree of automation for industries having significant investment. The descriptive statistics for the degree are presented in Table 5.

Table 5. Degree of automation for industries having significant investment

mean	*s.e.*	*min*	*max*
0.436	0.152	0.133	0.792

The average degree of automation was 0.436. The degree varies even among 25 industries with significant investment. The average degree of automation across these industries increased from 0.407 to 0.454 during the sample period. Therefore, we can observe that automation increased more rapidly over time in these industries than in other sectors with small investment. Figure 5 illustrates this increase in the average degree by the red line. Furthermore, during the sample period, the increase in the average degree across these 25 industries was more significant than that in the share of capital income. In Figure 9, assuming a Gaussian kernel function, we illustrate the kernel density of the automation degree using (32) in 1994 and 2020 for industries with significant investment. In these two years, the degree of automation varies among industries. The mode of the distribution was approximately 0.4. The density was approximately symmetric. Compared to the whole sample, from 1994 to 2020, the distribution moved more to the right with increased automation.

Finally, when we examined equation (35) with $NT = 675$, we found $d = 0.000448$, which was very small. In Figures 10.1 and 10.2, which correspond with 1994 and 2020, respectively, assuming a Gaussian kernel function, we illustrate the kernel density of the degree of automation for industries with significant investment using equations (32) and (34). The blue and red lines represent the degree of automation using equations (32) and (34), respectively. Compared with 1994, in 2020, the automation degree predicted the output level more precisely, similar to 54 industries.

5.4 Regression of the degree of automation

In this subsection, using the degree of automation, we explore how ICT capital, robotics stock, and R&D expenditure contribute to automation. We assume the following relationship between these three and the degree of automation:

$$\frac{a_{i,t}}{1-a_{i,t}} = c_i \left(\frac{v_{ai,t}}{v_{ai,0}}\right)^{\gamma_a} \left(\frac{v_{bi,t}}{v_{bi,0}}\right)^{\gamma_b} \left(\frac{v_{ci,t}}{v_{ci,0}}\right)^{\gamma_c}, \tag{36}$$

where $v_{ai,t}$, $v_{bi,t}$, and $v_{ci,t}$ are the ratio of ICT capital stock to labor, the ratio of robotics stock to labor, and the ratio of R&D expenditure to labor, respectively. Note that $\frac{a_{i,t}}{1-a_{i,t}}$ can take any positive value.

Taking the logarithm of equation (36), we examine the following panel regression with fixed effects:

$$\ln \frac{a_{i,t}}{1-a_{i,t}} = \beta_{c,i} + \gamma_a \ln \frac{va_{i,t}}{va_{i,0}} + \gamma_b \ln \frac{vb_{i,t}}{vb_{i,0}} + \gamma_c \ln \frac{vc_{i,t}}{vc_{i,0}} + \epsilon_{i,t}, \tag{37}$$

where $\epsilon_{i,t}$ is an error term which follows a distribution with mean zero and variance.

Given the wage rate to interest rate ratio, firms choose the degree of automation with the inputs of ICT capital stock, robotics stocks, R&D expenditure, and workers. Namely, in equation (37), the variables in the LHS and RHS may be correlated with each other. Therefore, we employ IV estimation for panel data. We use the following lagged variables for the instrumental variables: the ICT capital stock per labor unit, robotics stock per labor unit, R&D expenditure per labor unit, ICT capital return, R&D capital return, and the ratio of workers aged over fifty-five to the total workers. Regarding the ratio of workers over fifty-five to the total workers, we consider workers more easily replaced by machines than young workers. Workers over fifty-five are examined as older workers Acemoglu and Restrepo (2022) and Lerch (2022) presented evidence of the effect of automation on these older workers. We consider two estimations. We use one-period lagged instrumental variables in the first estimation and one- and two-period lagged instrumental variables in the second estimation.

Table 6. Estimation results of equation (37) ($\ln \frac{a_{i,t}}{1-a_{i,t}}$)

$variables$	(i)	(ii)
$ICT/labor$	0.0286	0.0268
	(8.59**)	(8.39**)
$Robots/labor$	0.0029	0.0050
	(1.35)	(2.84*)
$R\&D/labor$	0.0351	0.041
	(5.04**)	(6.07**)
$Underidentification\ test$	20.10**	23.06*
$Weak\ instrument\ tests$	334.12**	348.68**
$Overidentification\ test$	3.17	7.59

Note. Numbers in parentheses show the t-values by the wild bootstrap method. $*$ and $**$ represent significance at the 5% and 1% levels, respectively. In column (i), the sample size is 1386. The underidentification, weak instrument, and overidentification tests are distributed as the $\chi^2(4)$, $\chi^2(3)$, and $\chi^2(3)$, respectively. In column (ii), the sample size is 1333. the underidentification, weak instrument, and overidentification tests are distributed as the $\chi^2(10)$, $\chi^2(3)$, and $\chi^2(9)$, respectively.

Column (i) of Table 6 presents the estimation result using the one- and two-period lagged instrumental variables. Although the ICT capital stock per labor unit is significantly positive, the robotics stock per labor unit is insignificant. The ratio of ICT stock to capital stock, as well as the ratio of robotics to capital stock, has recently increased. During the sample period, the sectors' average ICT to capital stock ratio increased from 2.3 to 5.5%. The average ratio of robotics to capital stock increased from 0.4 to 0.95%. Even though this ratio of robotics was still small, the robotics stock was necessary for automation. However, the robotics stock fluctuated during the sample period in Japan. During the depression, many manufacturing firms moved production facilities abroad, especially to Asian economies. Therefore, confirming the strong impact of robotics on automation may be challenging. While the average ratio of R&D to capital stock among sectors increased from 20.0 to 22.7%, the R&D expenditure per labor unit is significantly positive. The underidentification test, which was a Cragg-Donald robust CUE-based one, showed no underidentification. The weak instrument test scrutinized the coefficients of three endogenous variables, $\gamma_j = 0$ ($j = a, b, c$), which convincingly rejected these null hypotheses. The overidentification test was two-step-GMM-based and showed that the orthogonality conditions were not rejected.

Column (ii) of Table 6 presents the estimation result using the one- and two-period lagged instrumental variables. Our study reveals a significant positive effect of robotics on the degree of automation. A sufficient number of instrumental variables supports this finding. The similarity of the results between columns (i) and (ii) further reinforces this conclusion, providing a clear and robust understanding of the impact of ICT and R&D on automation in the manufacturing sector.

6. Concluding remarks

This study proposed a method for measuring the degree of automation. First, by relating a task-based model to a CES production function, we demonstrate how the automation degree is involved in the CES production function. Next, examining panel data of Japanese manufacturing industries from 1994 to 2020, we estimated a CES production function. We calculated the automation degree assuming Pareto distributions of capital- and labor-input efficiency functions. The average of the automation degree across industries slowly increased from 0.411 to 0.428, similar to the share of capital income. When examining a subsample of industries with significant investment, we found a more considerable increase in automation from 0.407 to 0.454. In addition, using the whole sample, we also saw a significant impact of ICT capital, robotics stock, and R&D expenditure on the automation degree. Therefore, plausible results were obtained.

Our study raises several issues that warrant further discussion and potential future research. The first issue is the assumption of Pareto distributions about capital- and labor-input efficiency functions. Section 4.2 provides theoretical and practical justifications for this assumption, which is simple and tractable for measuring the degree of automation. However, there is still room for improvement, and we should consider several functions and choose the most appropriate one using a statistical criterion. Another issue is the difference between skilled and unskilled labor. We have observed a difference in automation between these two types of labor, suggesting the need to examine this difference by estimating a two-level CES production function.

Finally, increasing automation affects individuals' decisions not only about their work but also about their fertility and education investment for their children through their beliefs about technology replacing jobs in the future. These beliefs will change labor demand and supply now and in the future. Therefore, we need to know more about the degree of automation. We provide a simple and tractable method for measuring the degree of automation. This method can contribute to exploring the degree.

Appendix A. Proof of propositions and theorem

A.1 Proof of Proposition 1

First, we demonstrate that our production function is a neoclassical one. Given $\tau > 0$, the production function (13) is homogeneous of degree one:

$$\tau Y_t = F(\tau K_t, \tau L_t) = \Lambda(a(k_t))\tau L_t.$$

The marginal production of capital is

$$\frac{\partial Y_t}{\partial K_t} = \Theta'(a_t)a'(k_t)k_t + \Theta(a_t) = \Lambda'(a_t)a'(k_t) > 0,$$

because $\Lambda'(a_t) > 0$ and $a'(k_t) > 0$. The marginal product of labor is

$$\frac{\partial Y_t}{\partial L_t} = \Lambda(a_t) - \Lambda'(a_t)a'(k_t)k_t = \Theta'(a_t)a'(k_t)\frac{dk_t}{dL_t}K_t > 0,$$

because $\Theta'(a_t) < 0$, $a'(k_t) > 0$, and $\frac{dk_t}{dL_t} < 0$.

Using equations (8) and (12), it holds that

$$\frac{d^2\Lambda(a(k_t))}{dk_t^2} = \Lambda''(a_t)\left[a'(k_t)\right]^2 + \Lambda'(a_t)a''(k_t) = \left[\frac{1}{\Omega'(a_t)}\right]^2\left[\Lambda''(a_t) - \Lambda'(a_t)\frac{\Omega''(a_t)}{\Omega'(a_t)}\right]$$

$$= \left[\frac{1}{\Omega'(a_t)}\right]^2 \frac{\Lambda'(a_t)\left[-\Theta'(a_t)\right]\Lambda(a_t)}{\Lambda'(a_t)\Theta(a_t) - \Lambda(a_t)\Theta'(a_t)}\left\{\frac{\Lambda''(a_t)}{\Lambda'(a_t)} - \frac{\Theta''(a_t)}{\Theta'(a_t)} - 2\left[\frac{\Lambda'(a_t)}{\Lambda(a_t)} - \frac{\Theta'(a_t)}{\Theta(a_t)}\right]\right\}.$$

Note that

$$\Omega'(a_t) = \frac{\Lambda'(a_t)\Theta(a_t) - \Lambda(a_t)\Theta'(a_t)}{\Theta^2(a_t)},$$

$$\Omega''(a_t) = \frac{[\Lambda''(a_t)\Theta(a_t) - \Lambda(a_t)\Theta''(a_t)]\Theta^2(a_t) - [\Lambda'(a_t)\Theta(a_t) - \Lambda(a_t)\Theta'(a_t)]2\Theta(a_t)\Theta'(a_t)}{\Theta(a_t)^4},$$

$$\frac{\Omega''(a_t)}{\Omega'(a_t)} = \frac{\Lambda''(a_t)\Theta(a_t) - \Lambda(a_t)\Theta''(a_t)}{\Lambda'(a_t)\Theta(a_t) - \Lambda(a_t)\Theta'(a_t)} - 2\frac{\Theta'(a_t)}{\Theta(a_t)}.$$

The sign of $\frac{d^2\Lambda(a(k_t))}{dk_t^2}$ relies on

$$\frac{d\ln\frac{\Lambda'(a_t)}{\Theta'(a_t)}}{da_t} - 2\frac{d\ln\frac{\Lambda(a_t)}{\Theta(a_t)}}{da_t}.$$

Therefore, under equation (10), it holds that

$$\frac{d^2\Lambda(a(k_t))}{dk_t^2} = \Lambda''(a_t)\left(a'(k_t)\right)^2 + \Lambda'(a_t)a''(k_t) \leq 0.$$

Consequently, we obtain the following:

$$\frac{\partial^2 Y_t}{\partial K_t^2} = \left[\Lambda''(a_t)a'(k_t) + \Lambda'(a_t)a''(k_t)\right]\frac{1}{L_t} \leq 0,$$

$$\frac{\partial^2 Y_t}{\partial L_t^2} = \left[\Lambda''(a_t)(a'(k_t))^2 + \Lambda'(a_t)a''(k_t)\right](-k_t)\left(-\frac{K_t}{L_t^2}\right) \leq 0,$$

$$\frac{\partial^2 Y_t}{\partial K_t \partial L_t} = \left[\Lambda''(a_t)(a'(k_t))^2 + \Lambda'(a_t)a''(k_t)\right]\left(-\frac{K_t}{L_t^2}\right) \geq 0.$$

Second, we examine a neoclassical production function: $Y_t = F(K_t, L_t)$. Owing to the homogeneous degree one, it holds that

$$F\left(\frac{K_t}{Y_t}, \frac{L_t}{Y_t}\right) = 1.$$

The neoclassical production function can be represented by a production function in a task-based model when it holds that $\Theta(a_t) = \frac{K_t}{Y_t}$ and $\Lambda(a_t) = \frac{L_t}{Y_t}$.

A.2 Proof of Proposition 2

Output per labor unit is

$$y_t = \Lambda(a_t) = \left[[\Lambda(a_t)]^{-\rho}\right]^{-1/\rho}.$$

From equation (14),

$$y_t = \tau^{\frac{1}{\rho}} \left\{ b\left[\frac{\Theta(a_t)}{\Lambda(a_t)}\right]^{\rho} + (1-b) \right\}^{-1/\rho}.$$

Therefore, using equation (8), the production function in our task-based model is a CES production function. When $\rho = 0$, using equation (16), we have

$$\ln \Lambda(a_t) = \ln \tau + b \ln \frac{\Lambda(a_t)}{\Theta(a_t)}.$$

This implies a Cobb-Douglas production function,

$$y_t = \tau k_t^b.$$

Dividing a CES production function by Y_t, we obtain

$$1 = C\left[b\left(\frac{K_t}{Y_t}\right)^{-\rho} + (1-b)\left(\frac{L_t}{Y_t}\right)^{-\rho}\right]^{-\frac{1}{\rho}}$$

$$= C[b\Theta(a_t)^{\rho} + (1-b)\Lambda(a_t)^{\rho}]^{-\frac{1}{\rho}},$$

which implies equation (14). We examine when $\rho = 0$. Taking the logarithm of equation (14), we have

$$\frac{\ln[b\Theta(a_t)^{\rho} + (1-b)\Lambda(a_t)^{\rho}]}{-\rho} = \ln \tau. \qquad (A1)$$

The LHS numerator and denominator go to zero when $\rho \to 0$. Using the L'Hospital's rule, the LHS of equation (A1) equation implies equation (16):

$$\lim_{\rho \to 0} \frac{b\Theta(a_t)^\rho \ln \Theta(a_t) + (1-b)\Lambda(a_t) \ln \Lambda(a_t)}{b\Theta(a_t)^\rho + (1-b)\Lambda(a_t)^\rho}$$

$$= b \ln \Theta(a_t) + (1-b) \ln \Lambda(a_t) = \ln \tau.$$

A.3 Proof of Theorem 1

First, we examine when $\rho > 0$. Equation (19) is represented as

$$a_t = H_c(\Theta_t^{-1}) = 1 - \left(\frac{h_c}{\Theta_t^{-1}}\right)^\xi,$$

which implies

$$\Theta_t^{-1} = h_c(1-a_t)^{\frac{1}{\xi}}.$$

Using equation (14), it holds that

$$bh_c^{-\rho}(1-a_t)^{\frac{\rho}{\xi}} + (1-b)\Lambda_t^\rho = \tau. \tag{A2}$$

When $a_t \to 0$, the LHS of equation (A2) is $bh_c^{-\rho} = \tau$. Note that using the definition of Λ_t and Θ_t in equation (6),

$$\lim_{a_t \to 0} \Lambda_t = \lim_{a_t \to 0} \tau^{\frac{1}{\rho}} \left\{(1-b) + b\left(\frac{\Lambda_t}{\Theta_t}\right)^{-\rho}\right\}^{-\frac{1}{\rho}} = \lim_{a_t \to 0} \tau^{\frac{1}{\rho}} \left\{(1-b) + b\left[\frac{\int_0^{a_t} \theta(i)^{-1} di}{\int_{a_t}^1 \lambda(i)^{-1} di}\right]^{-\rho}\right\}^{-\frac{1}{\rho}}$$

$$= \tau^{\frac{1}{\rho}} \left\{(1-b) + b\left[\frac{0}{\int_0^1 \lambda(i)^{-1} di}\right]^{-\rho}\right\}^{-\frac{1}{\rho}} = 0.$$

Therefore, we obtain $h_c = \left(\frac{b}{\tau}\right)^{\frac{1}{\rho}}$. From equation (A2),

$$\tau(1-a_t)^{\frac{\rho}{\xi}} + (1-b)\Lambda_t^\rho = \tau.$$

This can be rewritten as follows:

$$1 - a_t = \left(1 - \frac{1-b}{\tau}\Lambda_t^\rho\right)^{\frac{\xi}{\rho}} = G_c(\Lambda_t^{-1}).$$

Because of the assumption about $G_c(\cdot)$ in equation (20), it holds that $\xi = \zeta = \rho$. Therefore, we obtain $H_c(\Theta_t^{-1})$ and $G_c(\Lambda_t^{-1})$ in equation (21).

Second, we examine when $\rho < 0$. Equation (19) is represented as

$$a_t = G_s(\Theta_t) = 1 - \left(\frac{g_s}{\Theta_t}\right)^\zeta,$$

which implies

$$\Theta_t = g_s a_t^{-\frac{1}{\zeta}}.$$

Using equation (14), it holds that

$$b g_s^\rho a_t^{-\frac{\rho}{\zeta}} + (1-b)\Lambda_t^\rho = \tau. \tag{A3}$$

When $a_t \to 1$, the LHS of equation (A3) is $b g_s^\rho = \tau$. Note that

$$\lim_{a_t \to 1} \Lambda_t = \lim_{a_t \to 0} \tau^{\frac{1}{\rho}} \left\{ (1-b) + b \left[\frac{\int_0^{a_t} \theta(i)^{-1} di}{\int_{a_t}^1 \lambda(i)^{-1} di} \right]^{-\rho} \right\}^{-\frac{1}{\rho}}$$

$$= \tau^{\frac{1}{\rho}} \left\{ (1-b) + b \left[\frac{\int_0^{a_t} \theta(i)^{-1} di}{0} \right]^{-\rho} \right\}^{-\frac{1}{\rho}} = \infty.$$

Therefore, we obtain $g_s = \left(\frac{b}{\tau}\right)^{-\frac{1}{\rho}}$. From equation (A3),

$$a_t = \left(1 - \frac{1-b}{\tau} \Lambda_t^\rho \right)^{-\frac{\zeta}{\rho}} = H_s(\Lambda_t).$$

Because of the assumption about $H_s(\cdot)$ in equation (20), it holds that $\xi = \zeta = -\rho$. Therefore, we obtain $H_s(\Theta_t)$ and $G_s(\Lambda_t)$ in equation (21).

A.4 Microfoundation of equation (21)

We examine equation (13). Owing to the homogeneous of degree one, it holds that $F\left(\frac{K_t}{Y_t}, \frac{L_t}{Y_t}\right) = \tau$, which can be rewritten as

$$F\left(\Theta(a_t)^{-1}, \Lambda(a_t)^{-1}\right) = \tau.$$

We examine the total differentiation:

$$F_1 \frac{d\Theta(a_t)^{-1}}{da_t} da_t + F_2 \frac{d\Lambda(a_t)^{-1}}{da_t} da_t = 0,$$

where $F_1 = \frac{\partial F(\cdot)}{\partial \Theta^{-1}}$ and $F_2 = \frac{\partial F(\cdot)}{\partial \Lambda^{-1}}$. It implies that

$$\frac{F_1}{F_2} = -\frac{d\Lambda(a_t)^{-1}}{d\Theta(a_t)^{-1}}.$$

Therefore, in equilibrium, the ratio of capital incomes to labor incomes is

$$\frac{R_t K_t / Y_t}{w_t L_t / Y_t} = \frac{F_1}{F_2} \frac{\Theta(a_t)^{-1}}{\Lambda(a_t)^{-1}} = -\frac{d\Lambda(a_t)^{-1} / \Lambda(a_t)^{-1}}{d\Theta(a_t)^{-1} / \Theta(a_t)^{-1}},$$

which implies

$$\frac{(R_t K_t)/Y_t}{(w_t L_t)/Y_t} = -\frac{d \ln \Lambda(a_t)^{-1}}{d \ln \Theta(a_t)^{-1}}. \tag{A4}$$

We examine a CES production function (15). In the following, to have a symmetric explanation of capital- and labor-input efficiency, we redefine $\Lambda^{-1}(a_t)$ as follows:

$$\Lambda^{-1}(\tilde{a}_t) = \int_0^{\tilde{a}_t} \lambda^{-1}(j)dj,$$

where $\tilde{a}_t = 1 - a_t$. Note that $\frac{L_t}{Y_t} = \Lambda^{-1}(a_t) = \Lambda^{-1}(\tilde{a}_t)$. Using equation (8), the LHS of equation (A4) is rewritten as

$$\frac{R_t K_t / Y_t}{w_t L_t / Y_t} = \frac{b}{\tilde{b}} \left(\frac{K_t}{L_t} \right)^{-\rho} = \frac{b}{\tilde{b}} \frac{(\Theta(a_t)^{-1})^{-\rho}}{(\Lambda(\tilde{a}_t)^{-1})^{-\rho}},$$

where $\tilde{b} = 1 - b$. Therefore,

$$-\frac{d \ln \Lambda(\tilde{a}_t)^{-1}}{d \ln \Theta(a_t)^{-1}} = \frac{(\Lambda(\tilde{a}_t)^{-1})^{\rho}/\tilde{b}}{(\Theta(a_t)^{-1})^{-\rho}/b}. \tag{A5}$$

We now explore the efficiency of capital and labor inputs, which satisfies equation (A5). We assume that

$$\frac{d \ln \Theta(a_t)^{-1}}{d a_t} = \xi \frac{(\Theta(a_t)^{-1})^{\rho}}{b}, \tag{A6}$$

where $\xi > 0$. Under equation (14), the assumption (A6) implies

$$\frac{d \ln \Lambda(\tilde{a}_t)^{-1}}{d \tilde{a}_t} = \xi \frac{(\Lambda(\tilde{a}_t)^{-1})^{\rho}}{\tilde{b}}.$$

We also introduce the parameter ν, which satisfies

$$\Theta(\nu) = \Lambda(\tilde{\nu}) = \tau^{\frac{1}{\rho}}, \tag{A7}$$

where

$$\nu = \{x | \Theta(x) = \Lambda(\tilde{x}), x \in [0, 1]\},$$

$\tilde{x} = 1 - x$ and $\tilde{\nu} = 1 - \nu$. Therefore, ν plays a role in normalization.

We examine the following integrals:

$$\int_{\Theta(\nu)^{-1}}^{\Theta(a_t)^{-1}} \frac{b}{\xi} \left[\Theta(x)^{-1} \right]^{-1-\rho} d\Theta(x)^{-1} = \int_{\nu}^{a_t} 1 dx$$

and

$$\int_{\Lambda(\tilde{\nu})^{-1}}^{\Lambda(\tilde{a}_t)^{-1}} \frac{\tilde{b}}{\xi} \left[\Lambda(x)^{-1} \right]^{-1-\rho} d\Theta(\tilde{x})^{-1} = \int_{\tilde{\nu}}^{\tilde{a}_t} 1 d\tilde{x},$$

where $\tilde{x} = 1 - x$. These imply

$$\left[-\frac{b}{\rho \xi} [\Theta(x)^{-1}]^{-\rho} \right]_{\Theta(\nu)^{-1}}^{\Theta(a_t)^{-1}} = a_t - \nu \quad and \quad \left[-\frac{\tilde{b}}{\rho \xi} [\Lambda(\tilde{x})^{-1}]^{-\rho} \right]_{\Lambda(\tilde{\nu})^{-1}}^{\Lambda(\tilde{a}_t)^{-1}} = \tilde{a}_t - \tilde{\nu}.$$

Therefore,

$$\frac{b}{\rho\xi}\Theta(a_t)^\rho = \frac{b}{\rho\xi}\Theta(\nu)^\rho - (a_t - \nu) \quad and \quad \frac{\tilde{b}}{\rho\xi}\Lambda(\tilde{a}_t)^\rho = \frac{\tilde{b}}{\rho\xi}\Lambda(\tilde{\nu})^\rho - (\tilde{a}_t - \tilde{\nu}).$$

Using $\Theta(\nu) = \Lambda(\tilde{\nu}) = \tau^{\frac{1}{\rho}}$, we obtain

$$\Theta(a_t) = \tau^{\frac{1}{\rho}}\left(1 - \rho\frac{\xi}{\tau}\frac{a_t - \nu}{b}\right)^{\frac{1}{\rho}} \quad and \quad \Lambda(\tilde{a}_t) = \tau^{\frac{1}{\rho}}\left(1 - \rho\frac{\xi}{\tau}\frac{\tilde{a}_t - \tilde{\nu}}{\tilde{b}}\right)^{\frac{1}{\rho}}.$$

Stipulating $\omega \equiv \rho\frac{\xi}{\tau}$, we assume that

$$\omega = \begin{cases} 1 & for \ \ \rho > 0, \\ -1 & for \ \ -1 \leq \rho < 0, \end{cases} \tag{A8}$$

where We also assume $\nu = \tilde{b}$ and b when $\rho > 0$ and $\rho < 0$, respectively. Consequently, when $\rho > 0$,

$$\Theta(a_t) = C\left(\frac{1 - a_t}{b}\right)^{\frac{1}{\rho}} \quad and \quad \Lambda(a_t) = C\left(\frac{a_t}{1 - b}\right)^{\frac{1}{\rho}}.$$

When $\rho < 0$,

$$\Theta(a_t) = C\left(\frac{a_t}{b}\right)^{\frac{1}{\rho}} \quad and \quad \Lambda(a_t) = C\left(\frac{1 - a_t}{1 - b}\right)^{\frac{1}{\rho}}.$$

Note that $C = \tau^{\frac{1}{\rho}}$.

A.5 Proof of Corollary 1

From equations (11), (17), and (21), we obtain (22).

A.6 Proof of Corollary 2

When $\rho > 0$,

$$\frac{\Lambda(a_t)}{\Theta(a_t)} = \left(\frac{a_t/(1 - b)}{(1 - a_t)/b}\right)^{\frac{1}{\rho}} = k_t.$$

When $-1 \leq \rho < 0$,

$$\frac{\Lambda(a_t)}{\Theta(a_t)} = \left(\frac{a_t/b}{(1 - a_t)/(1 - b)}\right)^{-\frac{1}{\rho}} = k_t.$$

These imply (23).

A.7 Proof of Corollary 3

Using equation (8), the degree of automation is

$$a_t = \frac{b(1 - b)(k_t^\rho - 1)}{b\rho + \rho(1 - b)k_t^\rho} + b. \tag{A9}$$

When $\rho \to 0$, it holds that

$$a_t = b(1-b)\ln k_t + b. \tag{A10}$$

Therefore, the automation degree in equations (A9) and (A10) has the continuity around $\rho = 0$. When $\rho = 0$, we also obtain

$$y_t = \Lambda(a_t) = c\exp\left(\frac{a_t - b}{1-b}\right),$$

which implies $y_t = ck_t^b$.

A.8 Microfoundation of equations (24) and (25)

In place of equation (A8), we assume that

$$\omega = \rho, \tag{A11}$$

which implies

$$\Theta(a_t) = C\left(1 - \rho\frac{a_t - \nu}{b}\right)^{\frac{1}{\rho}} \quad and \quad \Lambda(\tilde{a}_t) = C\left(1 - \rho\frac{\tilde{a}_t - \tilde{\nu}}{\tilde{b}}\right)^{\frac{1}{\rho}}. \tag{A12}$$

Assuming that $\nu = b$, we obtain (24) and (25). We can also assume another value of ν to satisfy $0 \le a_t \le 1$.

Appendix B. Other proofs

B.1 Proof of equation (10)

Taking the logarithm of equation (9), we have

$$\ln\frac{\lambda(a_t)}{\theta(a_t)} = 2\ln\frac{\Lambda(a_t)}{\Theta(a_t)} + \ln\frac{\Theta'(a_t)}{[-\Lambda'(a_t)]},$$

where $\zeta(a_t) \equiv \frac{\lambda(a_t)}{\theta(a_t)}$. Differentiating this by a_t, it holds that

$$\frac{d\ln\frac{\lambda(a_t)}{\theta(a_t)}}{da_t} = 2\left[\frac{\Lambda'(a_t)}{\Lambda(a_t)} - \frac{\Theta'(a_t)}{\Theta(a_t)}\right] + \frac{\Theta''(a_t)}{\Theta'(a_t)} - \frac{\Lambda''(a_t)}{\Lambda'(a_t)}.$$

Therefore, $\frac{d\frac{\lambda(a_t)}{\theta(a_t)}}{da_t} \ge 0$ is equal to equation (10):

$$\frac{\Theta'(a_t)}{\Theta(a_t)} - \frac{\Lambda'(a_t)}{\Lambda(a_t)} \ge \frac{1}{2}\left[\frac{\Theta''(a_t)}{\Theta'(a_t)} - \frac{\Lambda''(a_t)}{\Lambda'(a_t)}\right].$$

B.2 Average cost minimization in equation (11)

We examine

$$\min_{a_t \in [0,1]} \frac{R_t}{\Theta(a_t)} + \frac{w_t}{\Lambda(a_t)}.$$

The first-order condition is

$$R_t \left[-\frac{\Theta'(a_t)}{\Theta(a_t)^2} \right] + w_t \left[-\frac{\Lambda'(a_t)}{\Lambda(a_t)^2} \right] = 0.$$

This is equal to equation (9). Under equation (10), the second-order condition is

$$R_t \left\{ -\frac{\Theta''(a_t)\Theta(a_t)^2 - [\Theta'(a_t)]^2 2\Theta(a_t)}{\Theta(a_t)^4} \right\} + w_t \left\{ -\frac{\Lambda''(a_t)\Lambda(a_t)^2 - [\Lambda'(a_t)]^2 2\Lambda(a_t)}{\Lambda(a_t)^4} \right\}$$

$$= w_t \frac{\Lambda'(a_t)}{\Lambda(a_t)^2} \left\{ 2 \left[-\frac{\Theta'(a_t)}{\Theta(a_t)} + \frac{\Lambda'(a_t)}{\Lambda(a_t)} \right] + \frac{\Theta''(a_t)}{\Theta'(a_t)} - \frac{\Lambda''(a_t)}{\Lambda'(a_t)} \right\} \geq 0.$$

B.3 Proof of equation (17)

$$\frac{\partial Y_t}{\partial K_t} = C \left(\frac{Y_t}{C} \right)^{1+\rho} b K_t^{-\rho-1} = C^{-\rho} b \Theta_t^{1+\rho},$$

$$\frac{\partial Y_t}{\partial L_t} = C \left(\frac{Y_t}{C} \right)^{1+\rho} (1-b) L_t^{-\rho-1} = C^{-\rho} (1-b) \Lambda_t^{1+\rho}.$$

B.4 A CES production function with factor-augmenting technical progress

The output per labor unit is

$$y_t = A_{L,t} \Lambda(a_t) = A_{L,t} \left\{ [\Lambda(a_t)]^{-\rho} \right\}^{-1/\rho}.$$

Under equation (14), we have

$$y_t = A_{L,t} \left\{ b \left[\frac{\Theta(a_t)}{\Lambda(a_t)} \right]^{\rho} + (1-b) \right\}^{-1/\rho}.$$

Note that $\frac{\Lambda(a_t)}{\Theta(a_t)} = \frac{A_{K,t}}{A_{L,t}} k_t$. We obtain equation (27).

Appendix C. Degree of automation with the automation condition

C.1 Estimation of the automation condition

This appendix examines the degree of automation with the condition for automation adoption. The automation condition is

$$\frac{w_{i,t}}{R_{i,t}} = \frac{A_{L,t}}{A_{K,t}} \frac{\lambda(a_{i,t})}{\theta(a_{i,t})}.$$

We assume that $\frac{\lambda(a_{i,t})}{\theta(a_{i,t})}$ is an increasing function of $\frac{a_{i,t}}{1-a_{i,t}}$. We also assume (36). From the automation condition, the estimation equation is:

$$\ln \frac{w_{i,t}}{R_{i,t}} = \beta_{c,i} + \beta_t + \beta_a \ln \frac{v_{ai,t}}{v_{ai,0}} + \beta_b \ln \frac{v_{bi,t}}{v_{bi,0}} + \beta_c \ln \frac{v_{ci,t}}{v_{ci,0}} + \epsilon_{i,t}, \qquad (A13)$$

where $i = 1, \cdots, 54$ and $t = 1994,\ 1995, \cdots, 2020$. $\epsilon_{i,t}$ is an error term with a mean of zero and variance.

Given the wage rate to interest rate ratio, firms choose the degree of automation with the inputs of ICT capital stock, robotics stocks, and laborers. Namely, in equation (A13), the RHS includes the endogenous variables, $v_{ai,t}$, $v_{bi,t}$, and $v_{ci,t}$. Therefore, to account for the endogeneity of these three variables. We employ IV estimation for panel data. We use the following instrumental variables: the one-period lagged variables of the ICT capital stock per labor unit, robotics stock per labor unit, and R&D expenditure per labor unit. We also use the one-period lagged variables of the ICT capital return, the R&D capital return, and the ratio of workers aged over fifty-five to the total workers.

Table A1. Estimation results of equation (A13) ($\ln \frac{w_{i,t}}{R_{i,t}}$)

variable	*(i)*	*(ii)*
ICT/labor	0.0934	0.0949
	(2.05*)	(2.07*)
Robots/labor	0.0289	0.0288
	(2.24*)	(2.24*)
R&D/labor		−0.0003
		(−0.01)
Underidentification test	18.25**	17.16**
Weak instrument tests	11.67**	12.34**
Overidentification test	4.23	4.28

Note. The sample size is 1386. Numbers in parentheses show the t-values by the cluster-robust standard errors. $*$ and $**$ represent significance at the 5% and 1% levels, respectively. In column (i), the underidentification and weak instrument tests are distributed as the $\chi^2(3)$. The overidentification test is distributed as $\chi^2(2)$. In column (ii), the underidentification and weak instrument tests are distributed as the $\chi^2(4)$. The overidentification test is distributed as $\chi^2(3)$.

In the first estimation, the estimation results are presented in column named (i) of Table A1. The ICT capital stock per labor unit is significantly positive. The robotics stock per labor unit is also significantly positive. The ratio of ICT stock to capital stock, as well as the ratio of robotics to capital stock, has recently increased. Although these two capital stocks are still small in the aggregate capital stock, these two stocks are essential components for the automation condition. The three tests showed that the estimation might be appropriate. The results in the second estimation are presented in column (ii) of Table A1. Although ICT and robotics stocks were significantly positive, the impact of R&D expenditure was close to zero. In the automation condition, the R&D expenditure may not depend on the wage rate to interest rate ratio.

C.2 Degree of automation implied by the automation condition

Using the estimation result of the automation condition, we calculate the degree of automation. Under equation (21) with $\rho < 0$, the condition for automation adoption is:

$$\frac{w_{i,t}}{R_{i,t}} = \frac{A_{L,t}}{A_{K,t}} \frac{\lambda(a_{i,t})}{\theta(a_{i,t})} = \frac{A_{L,t}}{A_{K,t}} \left(\frac{b_i}{1 - b_i} \right)^{\frac{1}{-\rho}} \left(\frac{a_{i,t}}{1 - a_{i,t}} \right)^{\frac{1+\rho}{-\rho}}. \tag{A14}$$

Substituting equation (36) into equation (A14), we obtain the following relationships in equation (A13):

$$\beta_{c,i} = \frac{1+\rho}{-\rho} \ln c_i + \frac{1}{-\rho} \ln \frac{b_i}{1 - b_i} \quad and \quad \beta_j = \frac{1+\rho}{-\rho} \gamma_j, \quad j = a, b.$$

We did not examine R&D expenditure because it was insignificant in equation (A13).

The degree of automation is calculated as follows: $a_{i,t} = (1 + Z_{i,t}^{-1})^{-1}$, where $Z_{i,t} = \frac{a_{i,t}}{1-a_{i,t}}$ and

$$Z_{i,t} = \left(\frac{b_i}{1 - b_i} \right)^{\frac{-1}{1+\rho}} \exp \left(\beta_{c,i} \frac{-\rho}{1+\rho} \right) \left(\frac{v_{ai,t}}{v_{ai,0}} \right)^{\beta_a \frac{-\rho}{1+\rho}} \left(\frac{v_{bi,t}}{v_{bi,0}} \right)^{\beta_b \frac{-\rho}{1+\rho}}. \tag{A15}$$

The descriptive statistics for the automation degree are presented in Table A2. The sample size is 1386.

Table A2. Degree of automation implied by the automation condition

mean	*s.e.*	*min*	*max*
0.587	0.183	0.178	0.877

The average degree of automation was 0.59. Compared to the degree calculated by the aggregate capital stock, the degree was rather large. The average degree among industries increased little due to a small value of ρ in equation (A15).

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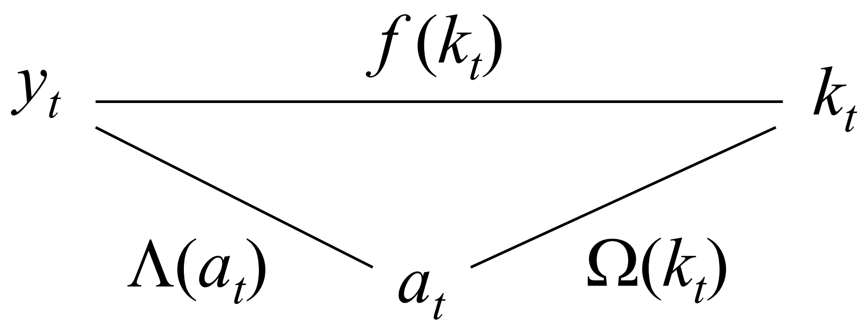

Figure 1. Decomposition of a production function

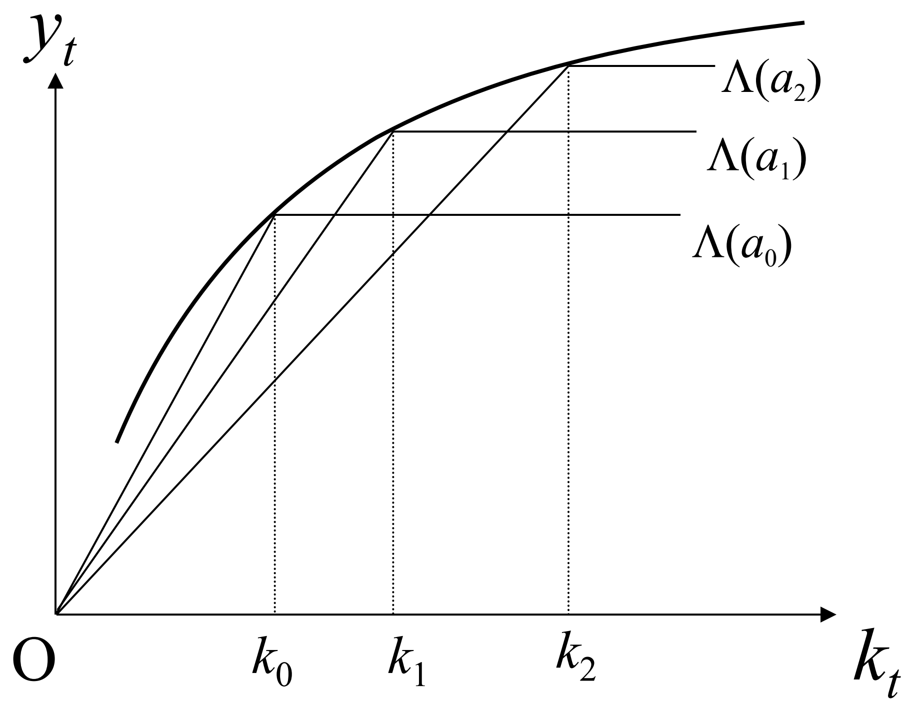

Figure 2. Production function through automation

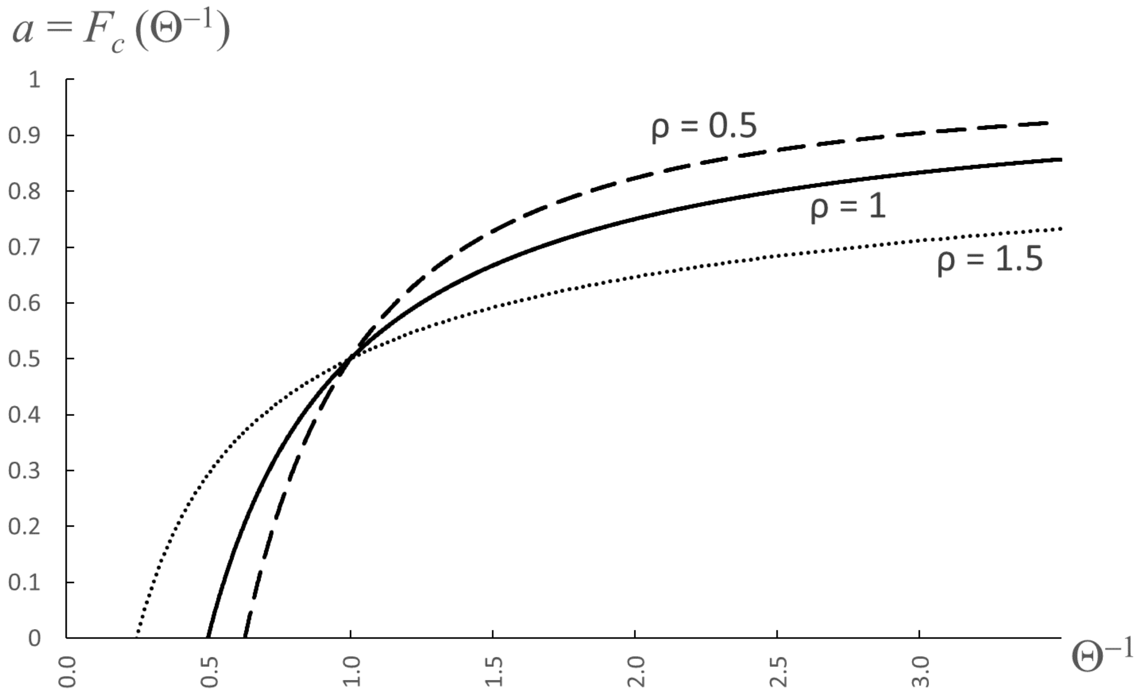

Figure 3.1 Degree of automation with a Pareto distribution when $\sigma < 1$

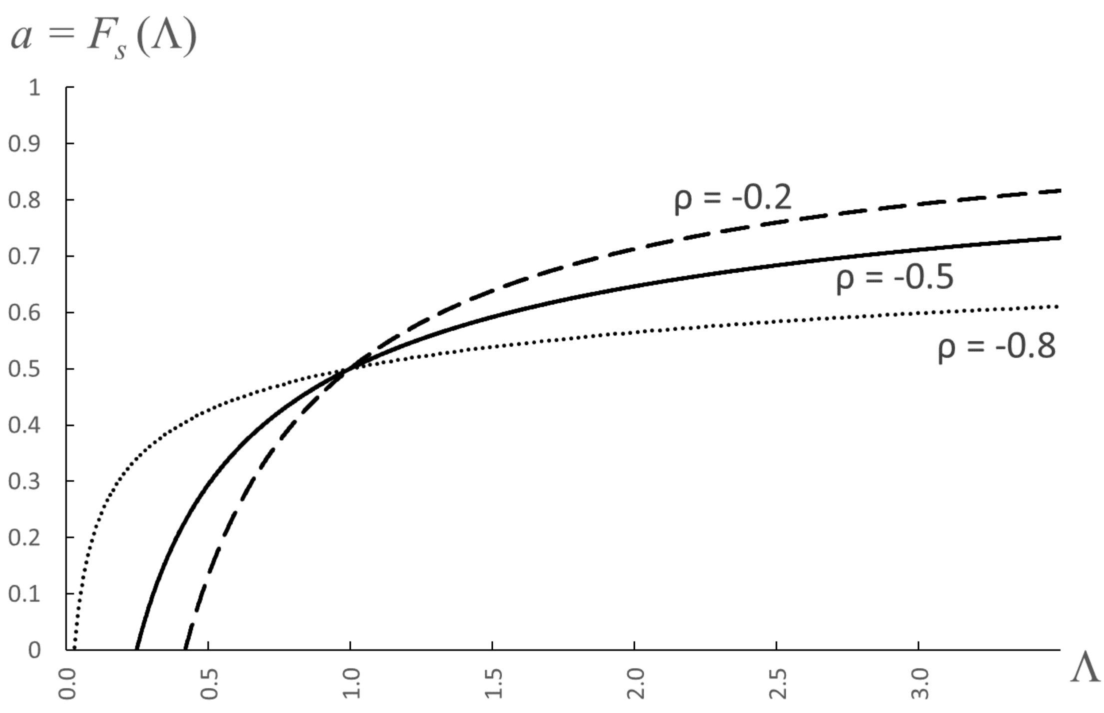

Figure 3.2 Degree of automation with a Pareto distribution when $\sigma > 1$

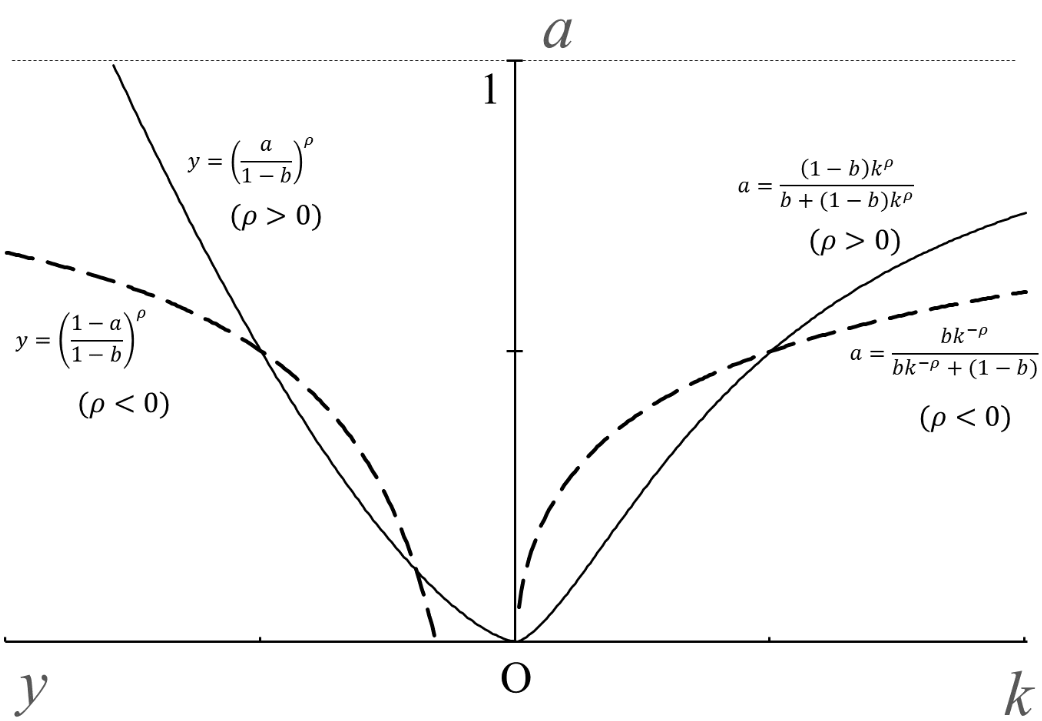

Figure 4. Relationships between y_t and a_t and between a_t and k_t

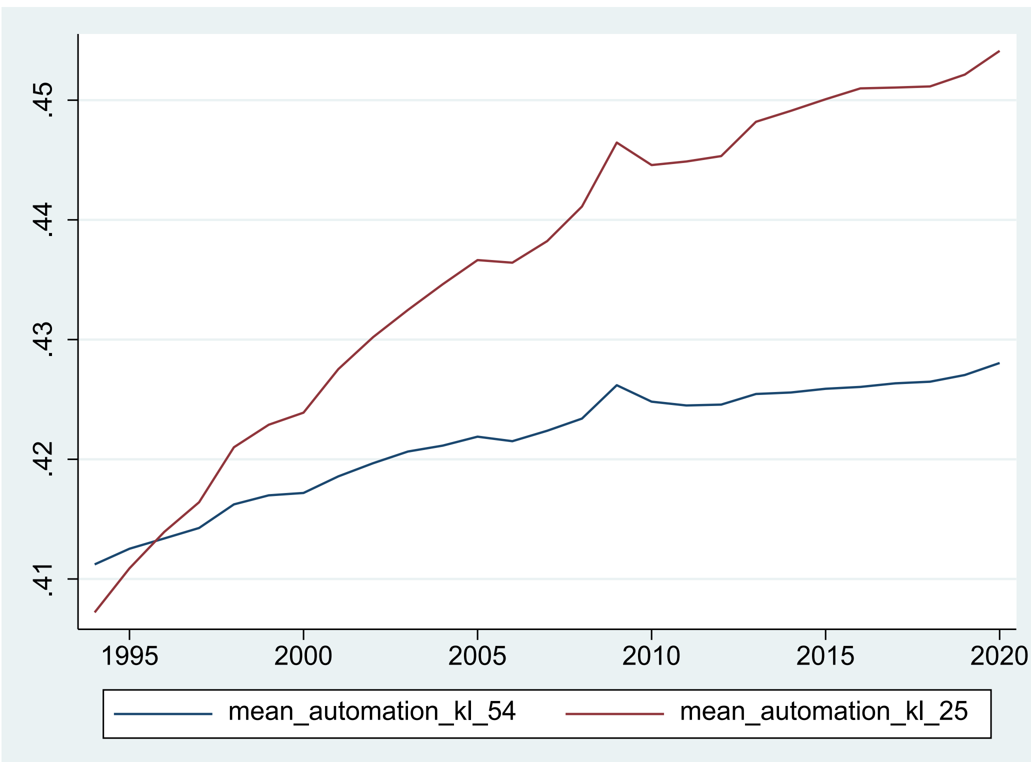

Figure 5. The average degree of automation across industries: 54 and 25 industries

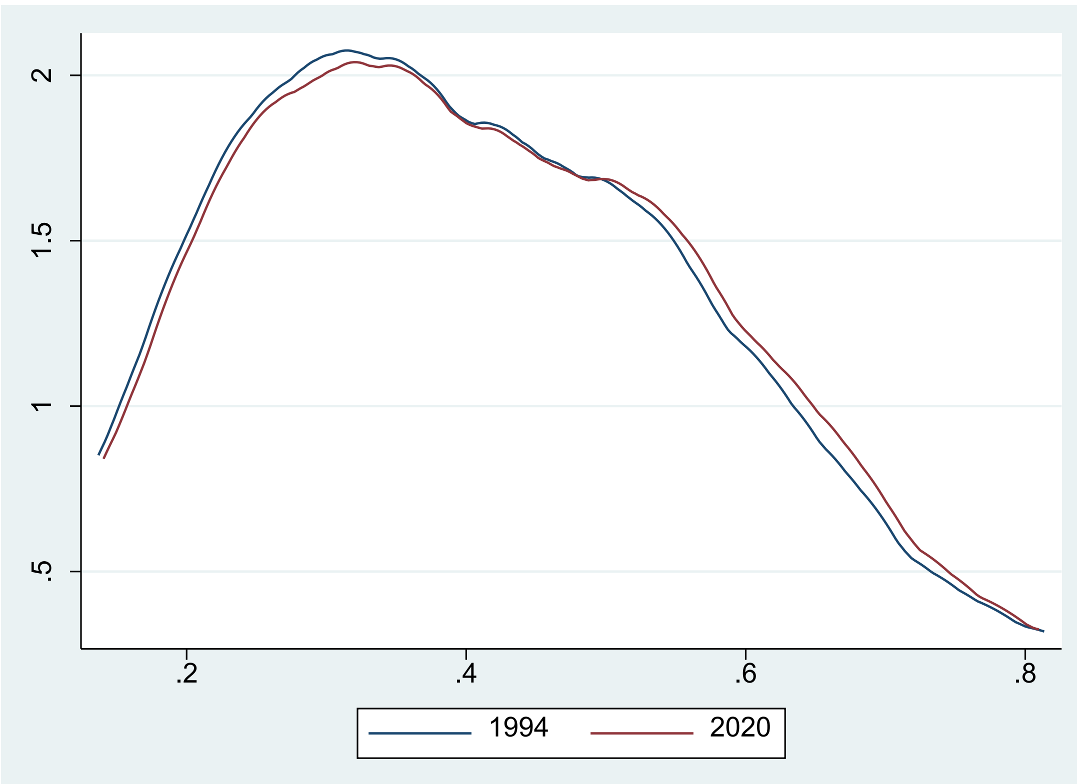

Figure 6. Kernel density of the automation degree in 1994 and 2020

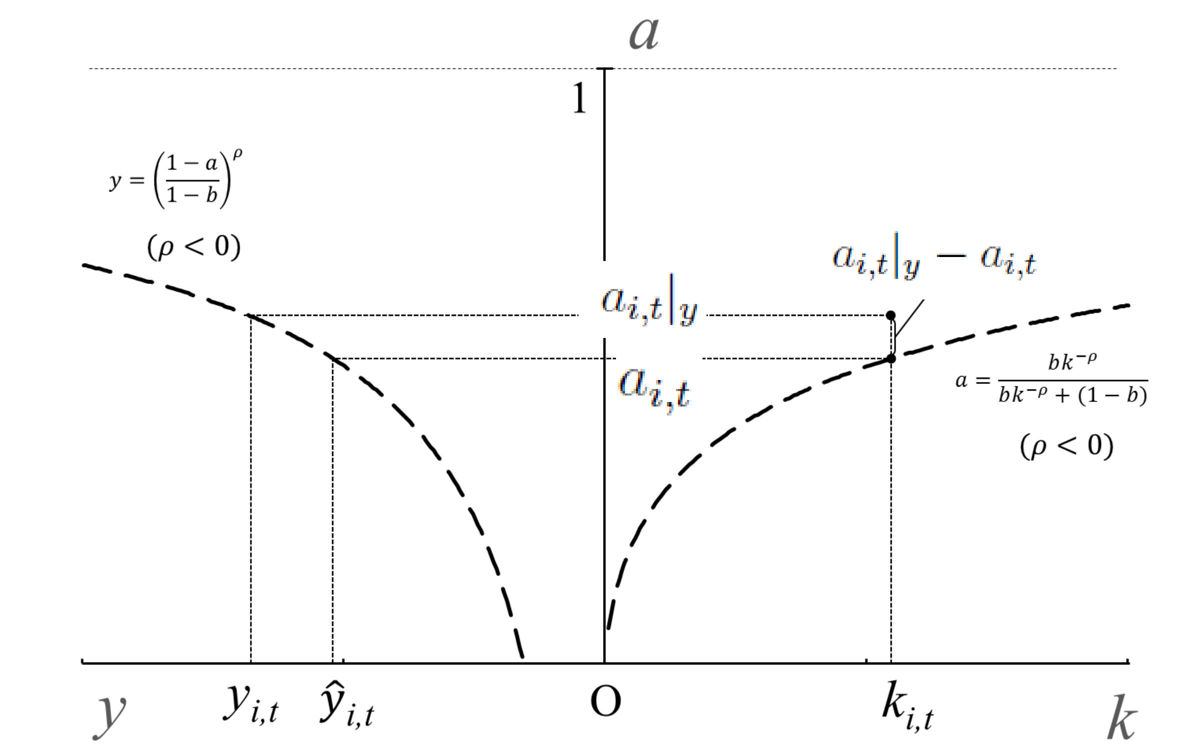

Figure 7. Gap in the degree of automation between equations (32) and (34)

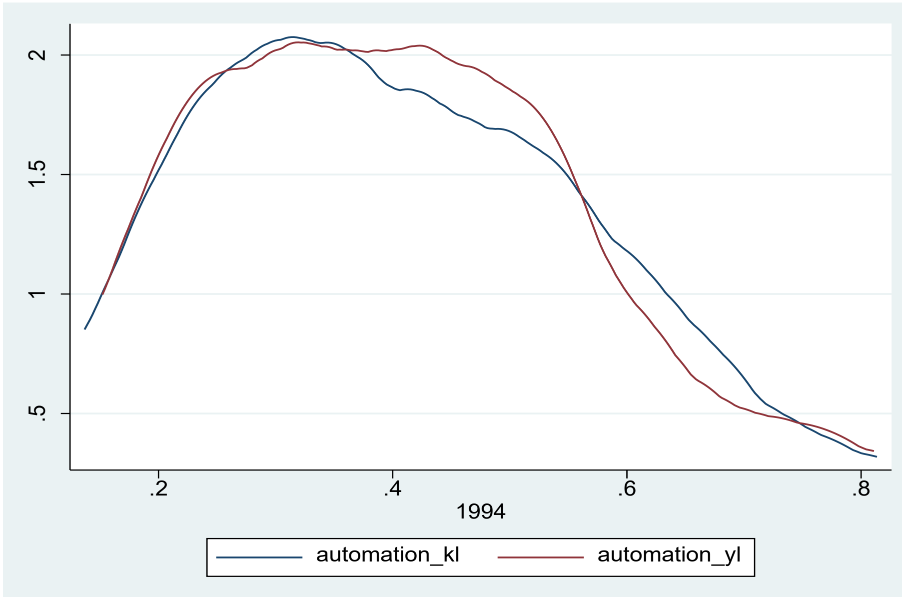

Figure 8.1 Kernel density of the automation degree in 1994: Comparing (32) with (34)

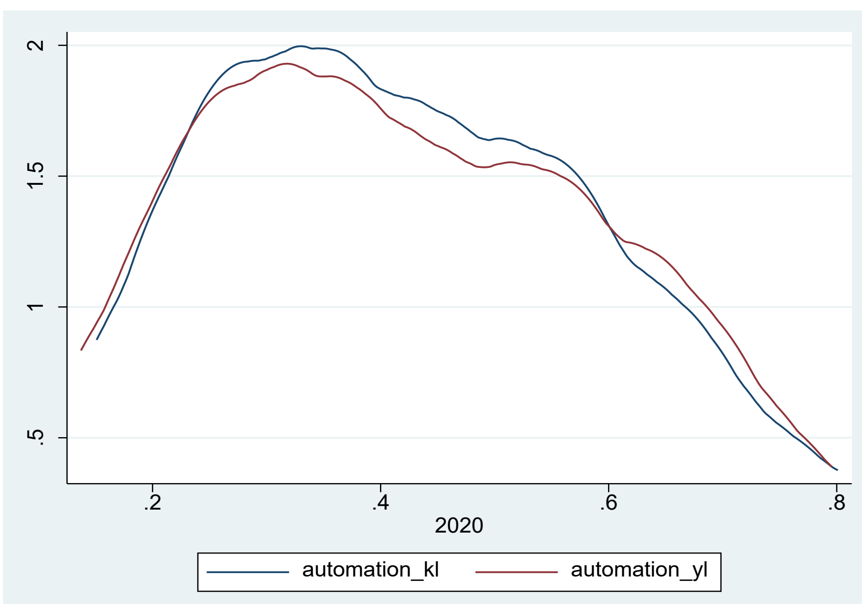

Figure 8.2 Kernel density of the automation degree in 2020: Comparing (32) with (34)

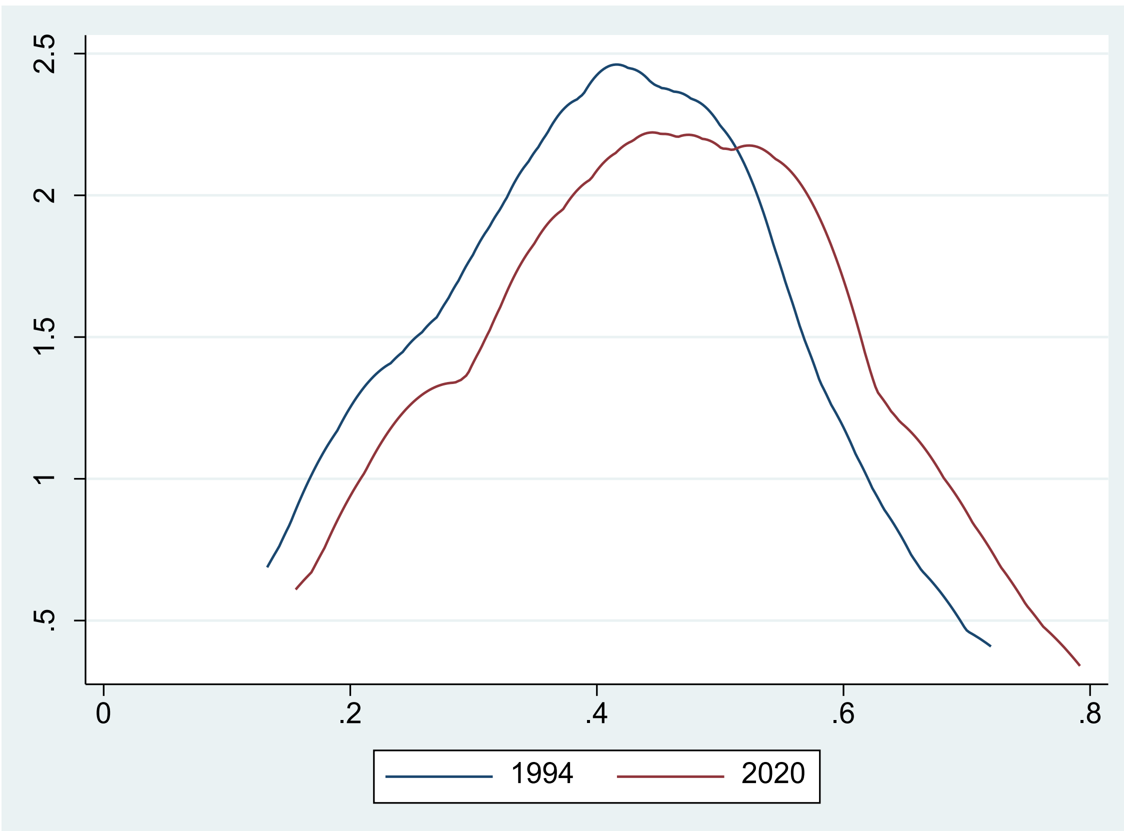

Figure 9. Kernel density of the automation degree in 1994 and 2020 for industries having significant investment

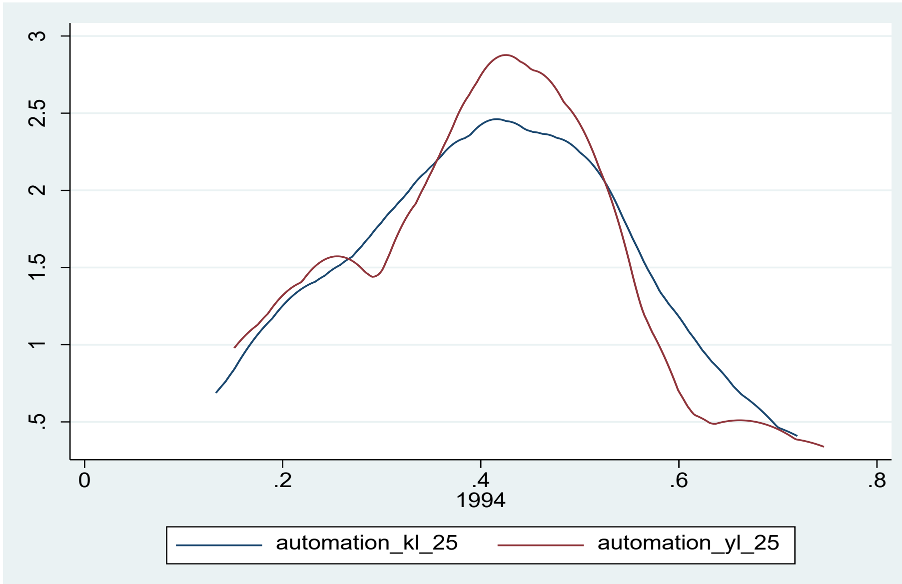

Figure 10.1 Kernel density of the automation degree in 1994 for industries having significant investment: Comparing (32) with (34)

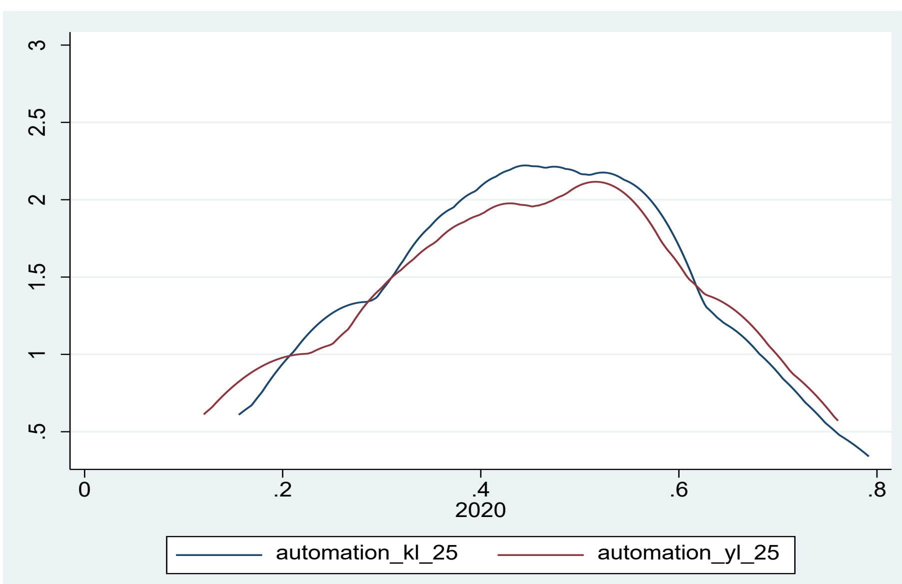

Figure 10.2 Kernel density of the automation degree in 2020 for industries having significant investment: Comparing (32) with (34)