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# Fiscal Rules and Economic Growth: A Politico-economic Approach

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# Fiscal Rules and Economic Growth: A Politico-economic Approach

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**Abstract**

This study constructs an overlapping generations model in which policies (capital income tax, labor income tax, public good provision, public investment, and public debt issuance) are determined by a probabilistic voting process. We characterize the politico-economic equilibrium in the absence and presence of fiscal rules. It is shown that the debt ceiling rule promotes economic growth and raises the welfare of future generations while lowering the welfare of the current generation. Further, the golden rule of public finance does not affect economic growth or the welfare of each generation.

- JEL Classification: D72, H41, H54, H63
- Keywords: Public investment, Fiscal rule, Probabilistic voting, Economic growth, Overlapping generations

## 1 Introduction

Governments finance the cost of public good provision and public investment through taxation and the issuance of public debt. Public good provision directly raises the level of individual welfare, and public investment significantly affects individual welfare through its contribution to economic growth. The effect of public investment on economic growth has been theoretically analyzed in many studies, such as Barro (1990), Futagami et al. (1993), Glomm and Ravikumar (1994), and Fisher and Turnovski (1998).[1] Aschauer (1989) and Bom and Ligthart (2014b) have empirically examined the economic growth-promoting effect of public investment. Meanwhile, public debt issuance slows down private capital accumulation and stagnates economic growth (Reinhart and Rogoff 2012; Checherita-Westphal and Rother 2012; Kumar and Woo 2015; Chudik et al. 2017).

Since the global financial crisis of 2008, many countries have experienced huge budget deficits and the ratios of public debt to GDP have remained elevated. Tax increases and reductions in government spending are necessary to maintain fiscal sustainability. However, policy packages with public investment reductions can reduce the productivity of the economy. In particular, the ratios of public investment to GDP have been declining in many OECD countries since 1970, and there is concern that underinvestment could stall economic growth.

Fiscal policy also creates intergenerational conflicts of interest. For example, public good provision benefits only the current generation, while public investment benefits both current and future generations. This creates a conflict of interest between the current and future generations in the allocation of resources between public good provision and public investment. In addition, since public debt issuance

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[1] Baxter and King (1993) and Baier and Glomm (2001) analyze situations in which governments implement both public goods provision and public investment.

shifts the tax burden from the current to future generations, intergenerational conflicts of interest also arise regarding public debt issuance.

Many countries are currently implementing various fiscal rules to improve fiscal sustainability. Typical fiscal rules include the balanced budget and debt ceiling rules; the latter sets a ceiling on outstanding public debt.[2] Both these rules can certainly enhance fiscal discipline and promote economic growth by mitigating the negative effect of public debt issuance on private capital accumulation (Azzimonti et al. 2016).[3] However, these rules may undermine intergenerational equity because they force the current generation to bear the cost of public investment that benefits future generations (Bom and Ligthart 2014a). In contrast, the golden rule of public finance allows only budget deficits to finance public investment, and thus, partially solves the problem associated with intergenerational equity. Several studies have constructed economic growth models with public capital, and shown that the golden rule of public finance can improve economic growth and the economic welfare of each generation (Greiner and Semmler 2000; Ghosh and Mourmouras 2004; Yakita 2008; Agenor and Yilmaz 2017; Maebayashi et al. 2017).

In democratic countries, economic policies are shaped through some political process. Many studies have developed politico-economic models in which policies are determined through the political process. For example, Song et al. (2012) analyze intergenerational political conflicts over public debt issuance. Arai et al. (2018) and Uchida and Ono (2021ab) investigate how fiscal rules affect politically determined fiscal policies, economic growth, and the welfare of each generation. Barseghyan and Battaglini (2016) examine the effects of austerity fiscal rules. This study seeks to contribute to this politico-economic literature. In particular, this study theoretically investigate (a) intergenerational political conflicts over fiscal policies, particularly public investment, (b) the pattern of economic growth on the politico-economic equilibrium, and (c) the impact of fiscal rules on economic growth and the welfare of each generation.

To analyze the above issues, this study constructs an overlapping generations model with private and public capital accumulation. Individuals are homogeneous within each generation and live for two periods (young and old). Firms produce a final good by using private capital and labor. Public investment raises the level of production technology. The government finances the cost of public good provision and public investment through income taxation and public debt issuance. Policies are determined through a political process. Following some studies (e.g., Song et al. 2012), we employ probabilistic voting to examine intergenerational conflicts of interest in the political process. We focus on Markov perfect politico-economic equilibria in which current policies depend on current state variables: private capital, public capital, and public debt. When voting, individuals take into account how current policies affect the state variables in the next period, and thus, affect policies in the next period.

We first assume the absence of any fiscal rule and show that there exists a Markov perfect politico-economic equilibrium in which labor income tax revenue, capital income tax revenue, public good provision, public investment, and public debt issuance are expressed as linear functions of output. We also analyze how changes in exogenous parameters, such as the population growth rate and political power of the old generation, affect the fiscal policies in the equilibrium. For example, we show that an increase in the population growth rate raises (lowers) the level of capital (labor) income tax paid by the old (young) generation. In addition, an increase in the population growth rate lowers the levels of public good provision and public debt issuance (thereby increasing fiscal discipline), and raises the level of public investment, which benefits only the young generation. Furthermore, this study derives comparative statics on the rate of economic growth, showing, for example, that higher population growth reduces the rate of economic growth.

We then analyze how fiscal rules affect economic growth patterns and each generation's welfare. This study focuses on two fiscal rules: the debt ceiling rule and golden rule of public finance. We

[2] For example, the Maastricht Treaty convergence criteria requires EU member countries to keep the ratio of public debt to GDP below 60%.

[3] Bisin et al. (2015) construct a politico-economic model of public debt where voters have time inconsistent preferences and present a new rationale for the balanced budget rule.

assume that, under the debt ceiling rule, the government faces the constraint that the ratio of public debt to private capital must be below a certain value. As in Arai et al. (2018) and Uchida and Ono (2021a), we show that the introduction of the debt ceiling rule raises the economic growth rate, and that the debt ceiling rule raises (lowers) the welfare of future generations (current generations). Finally, this study analyzes the characteristics of the politico-economic equilibrium under the golden rule of public finance. Under the simple setting in this study, even with no fiscal rule, the constraint that the amount of public debt issuance must be less than the amount of public investment does not bind. Thus, the politico-economic equilibria under the golden rule of finance and absence of the fiscal rule are identical. This implies that the golden rule of public finance does not affect economic growth patterns and each generation's welfare.

Here, we outline the differences between the most relevant studies and this study. Arai et al. (2018) present a politico-economic model with private capital accumulation, and investigate the effects of the debt ceiling rule on economic growth and economic welfare. However, they discard public capital accumulation, and thus, do not consider intergenerational conflicts over the allocation of resources between public investment and public good provision, nor do they analyze the effects of the golden rule of public finance. Baeseghyan and Battaglini (2016) build a politico-economic model in which public investment contributes to labor productivity growth and analyze the effects of austerity fiscal rules. However, their analysis is based on an infinite horizon model, so intergenerational conflicts of interest over economic policy are beyond consideration.

Uchida and Ono (2021ab), which are the most relevant to this study, construct overlapping generations models in which the level of public education, public good provision, income tax rates, and public debt issuance are politically determined. Uchida and Ono (2021a) analyze the effects of the debt ceiling rule, while Uchida and Ono (2021b) analyze the effects of the golden rule of public finance.[4] In these two studies, public education is regarded as public investment and human capital is accumulated through public education. Meanwhile, this study assumes a situation where public investment accumulates public capital and promotes economic growth. Furthermore, Uchida and Ono (2021b) adopt the golden rule of public finance in the form of "a certain percentage of public education expenditure must be financed by public debt issuance". However, this study adopts the golden rule of public finance in the form of "the amount of public debt issuance must be less than or equal to public investment spending, i.e., public investment may be financed entirely by taxation".

The rest of this article is organized as follows. Section 2 presents the model. Section 3 characterizes the politico-economic equilibrium in the absence of any fiscal rule. Section 4 characterizes the politico-economic equilibrium under the fiscal rules and investigates their effects. Section 5 presents concluding remarks.

# 2 Model

We consider a closed economy with overlapping generations. Individuals who are born in period $t$ $(\geq 0)$ are called generation $t$. They are homogeneous within each generation and live for two periods (youth and old age). The population grows at a constant rate of $n$ $(> -1)$:

$$N_{t+1} = (1+n)N_t,$$

where $N_t$ is the population size of generation $t$.

[4] Bassetto and Sargent (2006) construct an overlapping generations model with durable public goods and analyzes a situation where policies are determined through majority voting process under the golden rule of public finance; however, capital accumulation is not considered.

## 2.1 Individuals

Individuals obtain utility from the consumption of private and public goods in both periods, and their preferences are given by

$$\log c_t^y + \beta \log c_{t+1}^o + \theta \log g_t + \beta\theta \log g_{t+1}, \quad \beta \in (0,1), \quad \theta > 0,$$

where $c_t^y$ and $c_{t+1}^o$ are private good consumption when young and old, respectively, and $g_t$ and $g_{t+1}$ are per capita public good consumption when young and old, respectively. $\beta$ is the discount factor, and $\theta$ represents the degree of individuals' preferences for public good consumption.

When young, individuals supply one unit of labor inelastically, earn wages, and allocate their disposable income between private consumption and savings:

$$c_t^y + s_t \leq (1 - \tau_t^l) w_t, \tag{1}$$

where $s_t$ is savings, $w_t$ is a wage rate, and $\tau_t^l$ is a labor income tax rate, respectively. When old, they retire and consume the proceeds of savings:

$$c_{t+1}^o \leq (1 - \tau_{t+1}^k) R_{t+1} s_t, \tag{2}$$

where $R_{t+1}$ is a gross interest rate, and $\tau_{t+1}^k$ is a capital income tax rate.

Individuals choose private consumption and savings to maximize their utility, taking $w_t$, $R_{t+1}$, $\tau_t^l$, $\tau_{t+1}^k$, $g_t$, and $g_{t+1}$ as given. Solving the utility-maximization problem, we obtain

$$c_t^y = \frac{1}{1+\beta}(1 - \tau_t^l) w_t, \tag{3}$$

$$c_{t+1}^o = \beta(1 - \tau_{t+1}^k) R_{t+1} c_t^y, \tag{4}$$

$$s_t = \frac{\beta}{1+\beta}(1 - \tau_t^l) w_t. \tag{5}$$

In the initial period, each old individual, called generation $-1$, is endowed with $s_{-1}$ units of private physical capital (henceforth, private capital) and receives $(1 - \tau_0^k) R_0 s_{-1}$ units of return from savings. The utility function of individuals in generation $-1$ is given by $\log c_0 + \theta \log g_0$.

## 2.2 Firms

We assume a continuum of identical firms with a unit mass. They produce a final good by using private capital and labor as inputs. The production technology depends on public capital and is given by the following production function:

$$Y_t = A K_t^\alpha L_t^{1-\alpha} z_t^{1-\alpha}, \quad z_t \equiv \frac{Z_t}{L_t}, \quad A > 0, \quad \alpha \in (0,1),$$

where $Y_t$ is output, $K_t$ and $L_t$ are inputs of private capital and labor, respectively, $Z_t$ is public capital, $z_t$ is public capital per worker, $A$ is the productivity parameter, and $\alpha$ is a parameter representing the capital share in production.

All markets are perfectly competitive, and private capital depreciates within one period. Each firm chooses $K_t$ and $L_t$, taking $w_t$, $R_t$ and $z_t$ as given. The profit maximization conditions are given by

$$R_t = R(k_t, z_t) \equiv \alpha A k_t^{\alpha-1} z_t^{1-\alpha}, \tag{6}$$

$$w_t = w(k_t, z_t) \equiv (1-\alpha) A k_t^\alpha z_t^{1-\alpha}, \tag{7}$$

where $k_t \equiv K_t / L_t$ is private capital per worker.

## 2.3 The government

The government finances the cost of public good provision and public investment by income taxation and public debt issues. The budget constraint of the government is given by

$$N_t \tau_t^l w_t + N_{t-1} \tau_t^k R_t s_{t-1} + B_{t+1} = R_t B_t + G_t + I_t,$$

where $B_{t+1}$ is newly issued public debt, $G_t$ is public good provision, and $I_t$ is public investment, respectively. Capital and labor income tax revenues are represented by $N_{t-1}\tau_t^k R_{t-1} s_{t-1}$ and $N_t \tau_t^l w_t$, respectively. We denote public investment per worker by $i_t \equiv I_t / L_t$ and public debt per worker by $b_t \equiv B_t / L_t$. Noting that

$$G_t = (N_{t-1} + N_t) g_t \quad \Leftrightarrow \quad \frac{G_t}{N_t} = \frac{2+n}{1+n} g_t,$$

the budget constraint of the government is reformulated as

$$\tau_t^l w_t + \frac{1}{1+n} \tau_t^k R_t s_{t-1} + (1+n) b_{t+1} = R_t b_t + \frac{2+n}{1+n} g_t + i_t. \tag{8}$$

Public capital depreciates within one period, and the transition equation of public capital is given by

$$Z_{t+1} = I_t \quad \Leftrightarrow \quad z_{t+1} = F^z(i_t) \equiv \frac{1}{1+n} i_t. \tag{9}$$

## 2.4 Economic equilibrium

The market clearing condition for private capital is given by

$$K_{t+1} + B_{t+1} = N_t s_t.$$

Dividing both sides of the above condition by $N_t$ gives the following:

$$(1+n) k_{t+1} + (1+n) b_{t+1} = s_t. \tag{10}$$

From Equations (5), (6), (7), (8) and (10), we obtain the transition equations of private capital and public debt:

$$\begin{aligned} k_{t+1} &= F^k(\tau_t^k, \tau_t^l, g_t, i_t, k_t, z_t, b_t) \\ &\equiv \frac{1}{1+n} \left[ \frac{\beta}{1+\beta} w(k_t, z_t) + \frac{1}{1+\beta} \tau_t^l w(k_t, z_t) + \tau_t^k R(k_t, z_t)(k_t + b_t) - R(k_t, z_t) b_t - \frac{2+n}{1+n} g_t - i_t \right], \end{aligned} \tag{11}$$

$$\begin{aligned} b_{t+1} &= F^b(\tau_t^k, \tau_t^l, g_t, i_t, k_t, z_t, b_t) \\ &\equiv \frac{1}{1+n} \left[ R(k_t, z_t) b_t + \frac{2+n}{1+n} g_t + i_t - \tau_t^l w(k_t, z_t) - \tau_t^k R(k_t, z_t)(k_t + b_t) \right]. \end{aligned} \tag{12}$$

Equations (9), (11) and (12) show effects of public policies on capital and debt accumulation. First, higher capital and higher labor income tax rates promote private capital accumulation and reduce public debt issue: $\partial F^k / \partial \tau_t^k > 0$, $\partial F^k / \partial \tau_t^l > 0$, $\partial F^b / \partial \tau_t^k < 0$, and $\partial F^b / \partial \tau_t^l < 0$. Second, an increase in public good provision inhibits private capital accumulation and accelerates public debt accumulation: $\partial F^k / \partial g_t < 0$ and $\partial F^b / \partial g_t > 0$. Third, an increase in public investment slows down private capital accumulation, accelerates public debt accmulation and promotes public capital accumulation: $\partial F^k / \partial i_t < 0$, $\partial F^b / \partial i_t > 0$, and $dF^z / di_t > 0$.

# 3 The Politics

We consider the voting process on fiscal policies by employing probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. As shown in Persson and Tabellini (2000), the two candidates' platforms converge in the equilibrium to the same fiscal policies that maximize the weighted sum of the utility of voters.

## 3.1 Politico-economic equilibrium

We first consider a situation in which there is no constraint on public debt issues. From Equations (2), (3), (4), (6), (7) and (10), the welfare of the old, $V_t^o$, and that of the young, $V_t^y$, are given by

$$\begin{aligned} V_t^o &= V^o(\tau_t^k, g_t, k_t, z_t, b_t) \\ &\equiv \log(1-\tau_t^k) + \log R(k_t, z_t)(k_t + b_t) + \theta \log g_t, \end{aligned}$$

$$\begin{aligned} V_t^y &= V^y(\tau_t^l, g_t, \tau_{t+1}^k, g_{t+1}, k_t, z_t, k_{t+1}, z_{t+1}) \\ &\equiv (1+\beta)\log w(k_t, z_t) + (1+\beta)\log(1-\tau_t^l) + \theta \log g_t \\ &\quad + \beta \log(1-\tau_{t+1}^k) + \beta \log R(k_{t+1}, z_{t+1}) + \beta\theta \log g_{t+1}, \end{aligned}$$

where some irrelevant terms are omitted from the expressions. The political objective function, $\Omega_t$, is given by

$$\begin{aligned} \Omega_t &= \Omega(\tau_t^k, \tau_t^l, g_t, \tau_{t+1}^k, g_{t+1}, k_t, z_t, b_t, k_{t+1}, z_{t+1}) \\ &\equiv \omega V^o(\tau_t^k, g_t, k_t, z_t, b_t) + (1+n)(1-\omega)V^y(\tau_t^l, g_t, \tau_{t+1}^k, g_{t+1}, k_t, z_t, k_{t+1}, z_{t+1}) \\ &= \omega \log R(k_t, z_t)(k_t + b_t) + (1+n)(1-\omega)(1+\beta)\log w(k_t, z_t) \\ &\quad + \omega \log(1-\tau_t^k) + (1+n)(1-\omega)(1+\beta)\log(1-\tau_t^l) + \theta[\omega + (1+n)(1-\omega)]\log g_t \\ &\quad + \beta(1+n)(1-\omega)\log(1-\tau_{t+1}^k) + \beta(1+n)(1-\omega)\log R(k_{t+1}, z_{t+1}) + \beta\theta(1+n)(1-\omega)\log g_{t+1}, \end{aligned} \tag{13}$$

where $\omega \in [0,1]$ and $1-\omega$ are the relative weights on the old and young, respectively.

We focus on Markov-perfect politico-economic equilibria in which the size of fiscal policies depends on the current payoff-relevant state variables. In the present framework, these state variables are the private capital, $k_t$, public capital, $z_t$, and public debt, $b_t$. Thus, the capital income tax rate, $\tau_t^k$, labor income tax rate, $\tau_t^l$, public good provision, $g_t$, public investment, $i_t$, and public debt issue, $b_{t+1}$, are represented as functions of these state variables:

$$\tau_t^k = T_{NC}^k(k_t, z_t, b_t), \quad \tau_t^l = T_{NC}^l(k_t, z_t, b_t), \quad g_t = G_{NC}(k_t, z_t, b_t),$$
$$i_t = I_{NC}(k_t, z_t, b_t), \quad b_{t+1} = B_{NC}(k_t, z_t, b_t),$$

where the subscript "NC" implies that there is no constraint on public debt issues. Hereafter, we denote the next period's variable by a prime symbol: $x_{t+1} = x'$, $x = k, z, b, \tau^k$ and $g$.

**Definition 1.** *A Markov-perfect politico-economic equilibrium is a set of functions, $\{T_{NC}^k, T_{NC}^l, G_{NC}, I_{NC}, B_{NC}\}$, where $T_{NC}^k$ is a capital tax rate rule, $T_{NC}^l$ is a labor income tax rate rule, $G_{NC}$ is a public good provision rule, $I_{NC}$ is a public investment rule, and $B_{NC}$ is a public debt rule, so that $\{T_{NC}^k, T_{NC}^l, G_{NC}, I_{NC}, B_{NC}\}$ is a solution to the following problem:*

$$\begin{aligned} \max_{\tau^k, \tau^l, g, i, b'} \quad & \Omega(\tau^k, \tau^l, g, \tau^{k\prime}, g', k, z, b, k', z') \\ \text{subject to} \quad & k' = F^k(\tau^k, \tau^l, g, i, k, z, b) \\ & z' = F^z(i) \\ & b' = F^b(\tau^k, \tau^l, g, i, k, z, b) \\ & \tau^{k\prime} = T_{NC}^k(k', z', b') \\ & g' = G_{NC}(k', z', b'), \end{aligned} \tag{14}$$

We focus on a situation in which the after-tax capital income, $(1-\tau^k)R(k,z)(k+b)$, and public good provision, $g$, are represented as linear functions of per capita output, $Ak^\alpha z^{1-\alpha}$. In particular, we guess the capital income tax and public good provision rules such that

$$[1-T^k_{NC}(k,z,b)]\alpha A k^{\alpha-1} z^{1-\alpha}(k+b) = \delta^k_{NC} A k^\alpha z^{1-\alpha} \quad \Leftrightarrow \quad T^k_{NC}(k,z,b) = 1 - \frac{\delta^k_{NC}}{\alpha}\frac{k}{k+b}, \tag{15}$$

$$G_{NC}(k,z,b) = \delta^g_{NC} A k^\alpha z^{1-\alpha}, \tag{16}$$

where $\delta^k_{NC} > 0$ and $\delta^g_{NC} > 0$ are constant variables. The capital income tax rate function given by Equation (15) is decreasing in $k$ and increasing in $b$, while the public good provision function given by Equation (16) is increasing in $k$ and $z$.

Next, we investigate the characteristics of the interest rate, $R' = R(k',z')$, capital income tax rate in the next period, $\tau^{k\prime} = T^k_{NC}(k',z',b')$, and public good provision in the next period, $g' = G_{NC}(k',z',b')$, and then derive the Markov-perfect politico-economic equilibrium.

By differentiating $R'$ with respect to $\tau^k$, $\tau^l$, $g$, and $i$, we obtain

$$\frac{\partial R'}{\partial \tau^k} = \frac{\partial R}{\partial k'}\frac{\partial F^k}{\partial \tau^k} < 0, \quad \frac{\partial R'}{\partial \tau^l} = \frac{\partial R}{\partial k'}\frac{\partial F^k}{\partial \tau^l} < 0,$$

$$\frac{\partial R'}{\partial g} = \frac{\partial R}{\partial k'}\frac{\partial F^k}{\partial g} > 0, \quad \frac{\partial R'}{\partial i} = \frac{\partial R}{\partial k'}\frac{\partial F^k}{\partial i} + \frac{\partial R}{\partial z'}\frac{dF^z}{\partial i} > 0.$$

Increases in the current capital income tax rate, $\tau^k$, and current income tax rate, $\tau^l$, promote private capital accumulation, and thus, lower the interest rate, $R'$. By contrast, Increases in the current public good provision, $g$, and current public investment, $i$, hinder private capital accumulation, and thus, raise the interest rate, $R'$. Next, differentiating $\tau^{k\prime}$ with respect to $\tau^k$, $\tau^l$, $g$, and $i$ yields

$$\frac{\partial \tau^{k\prime}}{\partial \tau^k} = \frac{\partial T^k_{NC}}{\partial k'}\frac{\partial F^k}{\partial \tau^k} + \frac{\partial T^k_{NC}}{\partial b'}\frac{\partial F^b}{\partial \tau^k} < 0, \quad \frac{\partial \tau^{k\prime}}{\partial \tau^l} = \frac{\partial T^k_{NC}}{\partial k'}\frac{\partial F^k}{\partial \tau^k} + \frac{\partial T^k_{NC}}{\partial b'}\frac{\partial F^b}{\partial \tau^k} < 0,$$

$$\frac{\partial \tau^{k\prime}}{\partial g} = \frac{\partial T^k_{NC}}{\partial k'}\frac{\partial F^k}{\partial g} + \frac{\partial T^k_{NC}}{\partial b'}\frac{\partial F^b}{\partial g} > 0, \quad \frac{\partial \tau^{k\prime}}{\partial i} = \frac{\partial T^k_{NC}}{\partial k'}\frac{\partial F^k}{\partial i} + \frac{\partial T^k_{NC}}{\partial b'}\frac{\partial F^b}{\partial i} > 0.$$

Increases in the current capital income tax rate, $\tau^k$, and current labor income tax rate, $\tau^l$, lower the capital income tax rate in the next period, $\tau^{k\prime}$, because they stimulate private capital accumulation and reduce public debt issue. In contrast, Increases in the current public good provision, $g$, and current public investment, $i$, raise the capital income tax rate in the next period, $\tau^{k\prime}$, because they discourage private capital accumulation and accelerate public debt issue. Furthermore, differentiating $g'$ with respect to $\tau^k$, $\tau^l$, $g$, and $i$ yields

$$\frac{\partial g'}{\partial \tau^k} = \frac{\partial G_{NC}}{\partial k'}\frac{\partial F^k}{\partial \tau^k} > 0, \quad \frac{\partial g'}{\partial \tau^l} = \frac{\partial G_{NC}}{\partial k'}\frac{\partial F^k}{\partial \tau^l} > 0,$$

$$\frac{\partial g'}{\partial g} = \frac{\partial G_{NC}}{\partial k'}\frac{\partial F^k}{\partial g} < 0, \quad \frac{\partial g'}{\partial i} = \frac{\partial G_{NC}}{\partial k'}\frac{\partial F^k}{\partial i} + \frac{\partial G_{NC}}{\partial z'}\frac{dF^z}{di}.$$

Increases in the current capital income tax rate, $\tau^k$, and current labor income tax rate, $\tau^l$, increase the public good provision in the next period, $g'$, by promoting private capital accumulation. Meanwhile, an increase in the current public good provision, $g$, reduces the public good provision in the next period, $g'$, by inhibiting private capital accumulation.

The first-order conditions of the optimization problem in Equation (14) with respect to $\tau^k$, $\tau^l$, $g$, and $i$ are given by

$$\begin{aligned}
&\underbrace{-\frac{\beta(1+n)(1-\omega)}{1-\tau^{k\prime}}\frac{\partial \tau^{k\prime}}{\partial \tau^k}+\frac{\beta\theta(1+n)(1-\omega)}{g'}\frac{\partial g'}{\partial \tau^k}}_{MB_{\tau^k}}\\
&=\underbrace{\frac{\omega}{1-\tau^k}-\frac{\beta(1+n)(1-\omega)}{R'}\frac{\partial R'}{\partial \tau^k}}_{MC_{\tau^k}},
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\underbrace{-\frac{\beta(1+n)(1-\omega)}{1-\tau^{k\prime}}\frac{\partial \tau^{k\prime}}{\partial \tau^l}+\frac{\beta\theta(1+n)(1-\omega)}{g'}\frac{\partial g'}{\partial \tau^l}}_{MB_{\tau^l}}\\
&=\underbrace{\frac{(1+n)(1-\omega)(1+\beta)}{1-\tau^l}-\frac{\beta(1+n)(1-\omega)}{R'}\frac{\partial R'}{\partial \tau^l}}_{MC_{\tau^l}},
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\underbrace{\frac{\theta[\omega+(1+n)(1-\omega)]}{g}+\frac{\beta(1+n)(1-\omega)}{R'}\frac{\partial R'}{\partial g}}_{MB_g}\\
&=\underbrace{\frac{\beta(1+n)(1-\omega)}{1-\tau^{k\prime}}\frac{\partial \tau^{k\prime}}{\partial g}-\frac{\beta\theta(1+n)(1-\omega)}{g'}\frac{\partial g'}{\partial g}}_{MC_g},
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\underbrace{\frac{\beta(1+n)(1-\omega)}{R'}\frac{\partial R'}{\partial i}+\frac{\beta\theta(1+n)(1-\omega)}{g'}\frac{\partial G_{NC}}{\partial z'}\frac{dF^z}{di}}_{MB_i}\\
&=\underbrace{\frac{\beta(1+n)(1-\omega)}{1-\tau^{k\prime}}\frac{\partial \tau^{k\prime}}{\partial i}+\frac{\beta\theta(1+n)(1-\omega)}{g'}\frac{\partial G_{NC}}{\partial k'}\frac{\partial F^k}{\partial i}}_{MC_i}.
\end{aligned} \tag{20}$$

The left-hand sides of Equations (17) and (18) represent the marginal benefits of increasing the current capital and labor income tax rates, respectively. Increases in the current capital and labor income tax rates both lower the capital income tax rate in the next period and increase the public good provision in the next period, thus raising the welfare of the young. In contrast, the right-hand sides of Equations (17) and (18) represent the marginal costs of increasing the current capital and labor income tax rates, respectively. Higher tax rates not only increase the tax burden on the old and young but also reduce the welfare of the young by lowering the interest rate. The left-hand side of Equation (19) is the marginal benefit of increasing the current public good provision; such an increase directly raises the welfare of the old and young, and raises the welfare of the young by raising the interest rate. The right-hand side of Equation (19) is the marginal cost of increasing the current public good provision. Such an increase raises the capital income tax rate in the next period and reduces the public good provision in the next period, thus lowering the welfare of the young. The left and right sides of Equation (20) are the marginal benefit and marginal cost of increasing the current public investment, respectively. While an increase in the current public investment raises the interest rate and increases the public good provision in the next period through promoting public capital accumulation, it lowers the capital income tax rate in the next period and decreases the public good provision by inhibiting private capital accumulation.

By using Equations (17) - (20), we can verify the conjecture in Equations (15) and (16), and obtain the following result.

**Proposition 1.** *There is a Markov-perfect politico-economic equilibrium such that the policy functions* $T^k_{NC}$*,* $T^l_{NC}$*,* $G_{NC}$*,* $I_{NC}$*, and* $B_{NC}$ *are given by*

$$T^k_{NC}(k,z,b) = 1 - \frac{\omega}{\alpha(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}\frac{1}{1+b/k}, \tag{21}$$

$$T^l_{NC}(k,z,b) = \tau^l_{NC} \equiv 1 - \frac{(1+\beta)(1+n)(1-\omega)}{(1-\alpha)(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}, \tag{22}$$

$$G_{NC}(k,z,b) = \frac{\theta[\omega+(1+n)(1-\omega)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}\frac{1+n}{2+n}Ak^\alpha z^{1-\alpha}, \tag{23}$$

$$I_{NC}(k,z,b) = \frac{\beta(1-\alpha)(1+n)(1-\omega)}{\omega+(1+\beta)(1+n)(1-\omega)}Ak^\alpha z^{1-\alpha}. \tag{24}$$

$$B_{NC}(k,z,b) = \frac{\beta(1-\omega)[1-\alpha(1+\theta)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}Ak^\alpha z^{1-\alpha}. \tag{25}$$

*Proof.* See Appendix A.1. □

From Equations (21) - (24), the ratios of capital income tax revenue to output, $\tilde{T}^k_{NC}$, labor income tax revenue to output, $\tilde{T}^l_{NC}$, public good provision to output, $\tilde{G}_{NC}$ and public investment to output, $\tilde{I}_{NC}$, are respectively given by

$$\tilde{T}^k_{NC}\left(\frac{b}{k}\right) \equiv \frac{T^k_{NC}(k,z,b)R(k,z)(k+b)}{Ak^\alpha z^{1-\alpha}} = \alpha\left(1+\frac{b}{k}\right) - \frac{\omega}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}, \tag{26}$$

$$\tilde{T}^l_{NC} \equiv \frac{\tau^l_{NC}w(k,z)}{Ak^\alpha z^{1-\alpha}} = 1-\alpha-\frac{(1+\beta)(1+n)(1-\omega)}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}, \tag{27}$$

$$\tilde{G}_{NC} \equiv \frac{2+n}{1+n}\frac{G_{NC}(k,z,b)}{Ak^\alpha z^{1-\alpha}} = \frac{\theta[\omega+(1+n)(1-\omega)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}, \tag{28}$$

$$\tilde{I}_{NC} \equiv \frac{I_{NC}(k,z,b)}{Ak^\alpha z^{1-\alpha}} = \frac{\beta(1-\alpha)(1+n)(1-\omega)}{\omega+(1+\beta)(1+n)(1-\omega)}. \tag{29}$$

These policies have the following properties.

**Proposition 2.** Consider the Markov-perfect politico-economic equilibrium characterized in Proposition 1.

1. A higher ratio of public debt to private capital raises the ratio of capital income tax revenue to output; that is, $\partial\tilde{T}^k_{NC}/\partial(b/k) > 0$.

2. A higher rate of population growth raises the ratios of capital income tax revenue to output and public investment to output, and lowers the ratios of labor income tax revenue to output and public good provision to output; that is, $\partial\tilde{T}^k_{NC}/\partial n > 0$, $\partial\tilde{T}^l_{NC}/\partial n < 0$, $\partial\tilde{G}_{NC}/\partial n < 0$, and $\partial\tilde{I}_{NC}/\partial n > 0$.

3. A stronger preference for public good raises the ratios of capital income tax revenue to output, labor income tax revenue to output, and public good provision to output; that is, $\partial\tilde{T}^k_{NC}/\partial\theta > 0$, $\partial\tilde{T}^l_{NC}/\partial\theta > 0$, and $\partial\tilde{G}_{NC}/\partial\theta > 0$.

4. A higher relative political weight on the old raises the ratios of labor income tax revenue to output and public good provision to output, and lowers the ratios of capital income tax revenue to output and public investment to output; that is, $\partial\tilde{T}^k_{NC}/\partial\omega < 0$, $\partial\tilde{T}^l_{NC}/\partial\omega > 0$, $\partial\tilde{G}_{NC}/\partial\omega > 0$, and $\partial\tilde{I}_{NC}/\partial\omega < 0$.

*Proof.* The results are immediate from Equations (26) - (29). □

## 3.2 Economic growth

We proceed to investigate the pattern of economic growth. From Equations (9), (11), (12), (21), (22), (23), and (24), we obtain the transition equations of the state variables in the Markov perfect politico-economic equilibrium:

$$\begin{aligned} k_{t+1} &= F^k[T^k_{NC}(k_t, z_t, b_t), T^l_{NC}(k_t, z_t, b_t), G_{NC}(k_t, z_t, b_t), I_{NC}(k_t, z_t, b_t), k_t, z_t, b_t] \\ &= \frac{\alpha\beta(1-\omega)}{\omega + (1+\beta)(1+n)(1-\omega)} A k_t^\alpha z_t^{1-\alpha}, \end{aligned} \tag{30}$$

$$\begin{aligned} z_{t+1} &= F^z[I_{NC}(k_t, z_t, b_t)] \\ &= \frac{\beta(1-\alpha)(1-\omega)}{\omega + (1+\beta)(1+n)(1-\omega)} A k_t^\alpha z_t^{1-\alpha}, \end{aligned} \tag{31}$$

$$\begin{aligned} b_{t+1} &= F^b[T_{NC}(k_t, z_t, b_t), T^l_{NC}(k_t, z_t, b_t), G_{NC}(k_t, z_t, b_t), I_{NC}(k_t, z_t, b_t), k_t, z_t, b_t] \\ &= \frac{\beta(1-\omega)[1-\alpha(1+\theta)]}{(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]} A k^\alpha z^{1-\alpha}. \end{aligned} \tag{32}$$

From Equations (30) - (32), the ratios of public capital to private capital and public debt to private capital in period $t$ ($\geq 1$) are respectively given by

$$\frac{z_t}{k_t} = \chi^z_{NC} \equiv \frac{1-\alpha}{\alpha}, \tag{33}$$

$$\frac{b_t}{k_t} = \chi^b_{NC} \equiv \frac{1-\alpha(1+\theta)}{\alpha(1+\theta)}. \tag{34}$$

The ratios converge to $\chi^z_{NC}$ and $\chi^b_{NC}$ within one period, and thereafter, the stock of private capital, public capital, and public debt grow at the same rate. In other words, the economy reaches a balanced growth path (BGP). Furthermore, from Equations (30) and (33), the growth rate of private capital is obtained as

$$\frac{k_{t+1}}{k_t} = \begin{cases} \frac{\alpha\beta(1-\omega)A}{\omega+(1+\beta)(1+n)(1-\omega)} \left(\frac{z_0}{k_0}\right)^{1-\alpha} & \text{for} \quad t = 0 \\ \gamma_{NC} & \text{for} \quad t \geq 1 \end{cases} \tag{35}$$

where $\gamma_{NC}$ is the growth rate of private capital along the BGP, which is given by

$$\gamma_{NC} \equiv \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}\beta(1-\omega)A}{\omega + (1+\beta)(1+n)(1-\omega)}. \tag{36}$$

Let $y_t \equiv Y_t/N_t = Ak_t^\alpha z_t^{1-\alpha}$ denote per capita output. From Equations (30) and (31), we obtain $y_{t+1} = \gamma_{NC} y_t$ for $t \geq 0$, which implies that $y_t = \gamma^t_{NC} y_0$ for $t \geq 0$. The growth rate, $\gamma_{NC}$, has the following properties.

**Proposition 3.** *The growth rate along the BGP, $\gamma_{NC}$, is decreasing in the growth rate of population, $n$, and relative political weight on the old, $\omega$; that is, $\partial\gamma_{NC}/\partial n < 0$ and $\partial\gamma_{NC}/\partial\omega < 0$. Further, it is independent of the preference for public good; that is, $\partial\gamma_{NC}/\partial\theta = 0$.*

*Proof.* The results are immediate from Equation (36). □

The intuitive interpretation of Proposition 5 is as follows. First, dividing $k_{t+1}$ of (11) by $y_t = Ak_t^\alpha z_t^{1-\alpha}$ leads to

$$\frac{k_{t+1}}{Ak_t^\alpha z_t^{1-\alpha}} = \frac{1}{1+n}\left[\frac{\beta(1-\alpha)}{1+\beta} + \frac{1}{1+\beta}\tilde{T}^l_{NC} + \tilde{T}^k_{NC}\left(\frac{b_t}{k_t}\right) - \alpha\frac{b_t}{k_t} - \tilde{G}_{NC} - \tilde{I}_{NC}\right],$$

and the growth rate of private capital can be written as

$$
\begin{aligned}
\frac{k_{t+1}}{k_t} &= A\left(\frac{z_t}{k_t}\right)^{1-\alpha}\frac{k_{t+1}}{Ak_t^{\alpha}z_t^{1-\alpha}} \\
&= A\left(\frac{z_t}{k_t}\right)^{1-\alpha}\frac{1}{1+n}\left[\frac{\beta(1-\alpha)}{1+\beta}+\frac{1}{1+\beta}\tilde{T}^l_{NC}+\tilde{T}^k_{NC}\left(\frac{b_t}{k_t}\right)-\alpha\frac{b_t}{k_t}-\tilde{G}_{NC}-\tilde{I}_{NC}\right].
\end{aligned}
$$

Along the BGP, $z_t/k_t = \chi^z_{NC}$ and $b_t/k_t = \chi^b_{NC}$, and thus, the growth rate, $\gamma_{NC}$, is reformulated as

$$
\gamma_{NC} = A\left(\chi^z_{NC}\right)^{1-\alpha}\underbrace{\frac{1}{1+n}}_{\text{[GE.1]}}\left[\frac{\beta(1-\alpha)}{1+\beta}+\frac{1}{1+\beta}\underbrace{\tilde{T}^l_{NC}}_{\text{[GE.2]}}+\underbrace{\tilde{T}^k_{NC}\left(\chi^b_{NC}\right)}_{\text{[GE.3]}}-\underbrace{\alpha\chi^b_{NC}}_{\text{[GE.4]}}-\underbrace{\tilde{G}_{NC}}_{\text{[GE.5]}}-\underbrace{\tilde{I}_{NC}}_{\text{[GE.6]}}\right], \tag{37}
$$

where $\tilde{T}^k_{NC}\left(\chi^b_{NC}\right)$ is the ratio of capital income tax revenue to output along the BGP:

$$
\tilde{T}^k_{NC}\left(\chi^b_{NC}\right) = \frac{(1+\beta)(1+n)(1-\omega)}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}. \tag{38}
$$

The ratios of public debt to private capital, $\chi^b_{NC}$, and capital income tax revenue to output, $\tilde{T}^k_{NC}\left(\chi^b_{NC}\right)$, have the following properties.

**Lemma 1.** *The ratio of public debt to private capital along the BGP, $\chi^b_{NC}$, is decreasing in $\theta$, but is independent of $n$ and $\omega$; that is, $\partial\chi^b_{NC}/\partial n = 0$, $\partial\chi^b_{NC}/\theta < 0$, and $\partial\chi^b_{NC}/\partial\omega = 0$. Furthermore, the ratio of capital income tax revenue to output along the BGP, $\tilde{T}^k_{NC}(\chi^b_{NC})$, is increasing in $n$, and decreasing in $\theta$ and $\omega$; that is, $\partial\tilde{T}^k_{NC}(\chi^b_{NC})/\partial n > 0$, $\partial\tilde{T}^k_{NC}(\chi^b_{NC})/\partial\theta < 0$, and $\partial\tilde{T}^k_{NC}(\chi^b_{NC})/\partial\omega < 0$.*

*Proof.* The results are immediate from Equations (34) and (38). □

Using Equation (37) and Lemma 1, the properties of the economic growth rate along the BGP are summarized as follows.

First, a higher population growth rate has a private capital dispersing effect (see the term [GE.1]), lowers the ratio of labor income tax revenue to output (see the term [GE.2]), raises the ratio of capital income tax revenue to output (see the term [GE.3]), lowers the ratio of public good provision to output (see the term [GE.5]), and raises the ratio of public investment to output (see the term [GE.6]). In our setup, the sum of the first, second, and fifth effects dominates the sum of the third and fourth effects. Thus, higher population growth inhibits economic growth.

Next, a stronger preference for public good raises the ratio of labor income tax revenue to output (see the term [GE.2]), lowers the ratio of capital income tax revenue to output (see the term [GE.3]), lowers the ratio of public debt to private capital (see the term [GE.4]), and raises the ratio of public good provision to output (see the term [GE.5]). The rate of economic growth is independent of the strength of preference for public goods because the sum of the first and third effects completely cancels out the sum of the second and fourth effects.

Lastly, a higher relative political weight on the old raises the ratio of labor income tax revenue to output (see the term [GE.2]), lowers the ratio of capital income tax revenue to output (see the term [GE.3]), raises the ratio of public good provision to output (see the term [GE.5]), and lowers the ratio of public investment to output (see the term [GE.6]). The rate of economic growth is decreasing in the relative political weight on the old because the sum of the second and third effects dominates the sum of the first and fourth effects.

# 4 Fiscal Rules

In the previous section, we have investigated the characteristics of the political equilibrium with no constraint on public debt issues. However, in practice, many countries are trying to prevent the accumulation of public debt by implementing some fiscal rules. Here, we introduce two types of fiscal rules into the model, a debt-ceiling rule and a golden rule of public finance, and analyze their impact on politically determined fiscal policies, economic growth, and each generation's welfare.

## 4.1 Debt-ceiling rule

Some fiscal rules, such as the Maastricht Treaty convergence criteria, require governments to keep the ratio of public debt to GDP below certain levels. Some studies investigate the effects of a debt ceiling rule expressed as $B_{t+1}/Y_{t+1} = b_{t+1}/y_{t+1} \leq \eta$, where $\eta$ is a constant variable, on economic growth and welfare. Instead of the fiscal rule that $b_{t+1}/y_{t+1} \leq \eta$, this study considers a fiscal rule that the government must keep the ratio of public debt to private capital below a certain level; that is, $B_{t+1}/K_{t+1} = b_{t+1}/k_{t+1} \leq \eta$. By assuming such a fiscal rule, we can derive a political equilibrium in which the policy functions are expressed as simple functional forms.

Noting that $b'/k' = \chi^b_{NC}$ in the absence of the constraint on public debt issue and the debt-ceiling constraint that $b'/k' \leq \eta$ becomes binding if $\eta < \chi^b_{NC}$, we obtain the following proposition. The policy functions under the debt-ceiling rule are denoted by the subscript "DC".

**Proposition 4.** *If $\eta < \chi^b_{NC}$, the debt ceiling constraint is binding and the policy functions are given by*

$$T^k_{DC}(k,z,b) = 1 - \frac{\omega}{\alpha(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}\frac{1}{1+b/k}, \tag{39}$$

$$\begin{aligned} T^l_{DC}(k,z,b) &= \tau_{DC} \\ &\equiv \frac{(1+\beta)(1+\eta)}{(1-\alpha)1+\beta+\eta}\left\{\frac{\theta[\omega+(1+n)(1-\omega)]+\omega+\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]} - \alpha\right\} \\ &\quad - \frac{1+\beta}{1+\beta+\eta}\frac{\beta\eta}{1+\beta}. \end{aligned} \tag{40}$$

$$G_{DC}(k,z,b) = \frac{\theta[\omega+(1+n)(1-\omega)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}\frac{1+n}{2+n}Ak^\alpha z^{1-\alpha}, \tag{41}$$

$$I_{DC}(k,z,b) = \frac{\beta(1-\alpha)(1+n)(1-\omega)}{\omega+(1+\beta)(1+n)(1-\omega)}Ak^\alpha z^{1-\alpha}, \tag{42}$$

$$B_{DC}(k,z,b) = \frac{\beta}{1+\beta+\eta}\frac{(1-\omega)[1+\alpha\beta(1+\theta)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}Ak^\alpha z^{1-\alpha}. \tag{43}$$

*If $\eta \geq \chi^b_{NC}$, the debt-ceiling constraint is non-binding, and the policy functions are identical to those in the absence of constraint on public debt issues.*

*Proof.* See Appendix A.2. □

Hereafter, we assume that $\eta < \chi^b_{NC}$. In this case, the debt-ceiling constraint is binding, and the ratio of public debt to private capital in period $t$ ($\geq 1$) is given by $b_t/k_t = \eta$. From Equations (9), (11), (39), (40), (41), and (42), the transition equations of private capital and public capital under the debt-ceiling fiscal rule are respectively represented as

$$\begin{aligned} k_{t+1} &= F^k[T^k_{DC}(k_t,z_t,b_t), T^l_{DC}(k_t,z_t,b_t), G_{DC}(k_t,z_t,b_t), I_{DC}(k_t,z_t,b_t), k_t, z_t, b_t] \\ &= \frac{\beta(1-\omega)[1+\alpha\beta(1+\theta)]}{(1+\theta)(1+\beta+\eta)[\omega+(1+\beta)(1+n)(1-\omega)]}Ak_t^\alpha z_t^{1-\alpha}, \end{aligned} \tag{44}$$

$$\begin{aligned} z_{t+1} &= F^z[I_{DC}(k_t,z_t,b_t)] \\ &= \frac{\beta(1-\alpha)(1-\omega)}{\omega+(1+\beta)(1+n)(1-\omega)}Ak_t^\alpha z_t^{1-\alpha}, \end{aligned} \tag{45}$$

and the ratio of public capital to private capital in period $t$ ($\geq 1$) is given by

$$\frac{z_t}{k_t} = \chi^z_{DC} \equiv \frac{(1-\alpha)(1+\theta)(1+\beta+\eta)}{1+\alpha\beta(1+\theta)}. \tag{46}$$

The ratios of public capital to private capital and public debt to private capital converge to $\chi^z_{DC}$ and $\eta$, respectively. Thereafter, the economy reaches a BGP. Furthermore, the growth rate of private capital is obtained as

$$\frac{k_{t+1}}{k_t} = \begin{cases} \frac{\beta(1-\omega)[1+\alpha\beta(1+\theta)]A}{(1+\theta)(1+\beta+\eta)[\omega+(1+\beta)(1+n)(1-\omega)]}\left(\frac{z_0}{k_0}\right)^{1-\alpha} & \text{for} \quad t=0 \\ \gamma_{DC} & \text{for} \quad t \geq 1 \end{cases} \tag{47}$$

where $\gamma_{DC}$ is the growth rate of private capital along the BGP, which is given by

$$\gamma_{DC} \equiv \frac{\beta(1-\alpha)^{1-\alpha}(1-\omega)[1+\alpha\beta(1+\theta)]^{\alpha}A}{(1+\theta)^{\alpha}(1+\beta+\eta)^{\alpha}[\omega+(1+\beta)(1+n)(1-\omega)]}. \tag{48}$$

From Equations (44) and (45), we obtain $y_{t+1} = \gamma_{DC} y_t$ for $t \geq 0$, which implies that $y_t = \gamma^t_{DC} y_0$ for $t \geq 0$.

**Proposition 5.** *The growth rate along the BGP, $\gamma_{DC}$, is decreasing in the growth rate of population, $n$, degree of public good preference, $\theta$, and relative political weight on the old, $\omega$; that is, $\partial\gamma_{DC}/\partial n < 0$, $\partial\gamma_{DC}/\partial\theta < 0$, and $\partial\gamma_{DC}/\partial\omega < 0$.*

*Proof.* Differentiating $\gamma_{DC}$ of Equation (48) with respect to $n$, $\theta$, and $\omega$, respectively gives the results. □

## 4.2 Effects of the debt-ceiling rule

We analyze the impact of the debt-ceiling rule on the politically determined fiscal policies and economic growth patterns. By comparing the politico equilibrium in the presence of the debt-ceiling rule with that in the absence of the fiscal rule, we obtain the following lemma.

**Lemma 2.** *The policy functions of capital income tax rate, public good provision, and public investment under the debt-ceiling rule are identical to those in the absence of the fiscal rule:*

$$T^k_{DC}(k,z,b) = T^k_{NC}(k,z,b), \quad G_{DC}(k,z,b) = G_{NC}(k,z,b), \quad I_{DC}(k,z,b) = I_{NC}(k,z,b).$$

*Furthermore, if $\eta < \chi^b_{NC}$, the labor income tax rate in the presence of the debt-ceiling rule is higher than that in the absence of the fiscal rule:*

$$T^l_{DC}(k,z,b) = \tau^l_{DC} > \tau^l_{NC} = T^l_{NC}(k,z,b).$$

*Proof.* See Appendix A.3. □

Lemma 2 implies that the ratios of public good provision to output and public investment to output under the debt-ceiling rule are identical to those in the absence of the fiscal rule:

$$\frac{G_{DC}(k,z,b)}{Ak^{\alpha}z^{1-\alpha}} = \tilde{G}_{DC} = \tilde{G}_{NC} = \frac{G_{NC}(k,z,b)}{Ak^{\alpha}z^{1-\alpha}},$$

$$\frac{I_{DC}(k,z,b)}{Ak^{\alpha}z^{1-\alpha}} = \tilde{I}_{DC} = \tilde{I}_{NC} = \frac{I_{NC}(k,z,b)}{Ak^{\alpha}z^{1-\alpha}}.$$

Further, in the case where $\eta < \chi^b_{NC}$, the ratio of labor income tax revenue to output under the debt-ceiling rule is higher than that in the absence of the fiscal rule:

$$\frac{\tau^l_{DC}w(k,z)}{Ak^{\alpha}z^{1-\alpha}} = \tilde{T}^l_{DC} > \tilde{T}^l_{NC} = \frac{\tau^l_{NC}w(k,z)}{Ak^{\alpha}z^{1-\alpha}}.$$

Lemma 2 also implies that, for a given level of $b/k$, the ratio of capital income tax revenue to output under the debt-ceiling rule is equal to that in the absence of the fiscal rule:

$$\frac{T_{DC}^k(k,z,b)R(k,z)(k+b)}{Ak^\alpha z^{1-\alpha}} = \tilde{T}_{DC}^k\left(\frac{b}{k}\right) = \tilde{T}_{NC}^k\left(\frac{b}{k}\right) = \frac{T_{NC}^k(k,z,b)R(k,z)(k+b)}{Ak^\alpha z^{1-\alpha}}.$$

Noting that $\tilde{T}_{DC}^k$ (and $\tilde{T}_{NC}^k$) is increasing with respect to $b/k$, when $\eta < \chi_{NC}^b$, the introduction of the debt-ceiling rule lowers the ratio of capital income revenue to output along the BGP; that is, $\tilde{T}_{DC}^k(\eta) < \tilde{T}_{NC}^k(\chi_{NC}^b)$. Furthermore, comparing $\chi_{NC}^z$ of Equation (33) with $\chi_{DC}^z$ of Equation (46) yields $\chi_{DC}^z < \chi_{DC}^z$.

We next investigate how the introduction of the debt-ceiling rule affects the economic growth rate along the BGP. The growth rate along the BGP under the debt-ceiling rule, $\gamma_{DC}$, given by Equation (48), and that in the absence of the fiscal rule, $\gamma_{NC}$, given by Equation (36), are respectively reformulated as

$$\gamma_{DC} = A(\chi_{DC}^z)^{1-\alpha}\frac{1}{1+n}\left[\frac{\beta(1-\alpha)}{1+\beta} + \frac{1}{1+\beta}\tilde{T}_{DC}^l + \tilde{T}_{DC}^k(\eta) - \alpha\eta - \tilde{G}_{DC} - \tilde{I}_{DC}\right],$$

$$\gamma_{NC} = A(\chi_{NC}^b)^{1-\alpha}\frac{1}{1+n}\left[\frac{\beta(1-\alpha)}{1+\beta} + \frac{1}{1+\beta}\tilde{T}_{NC}^l + \tilde{T}_{NC}^k(\chi_{NC}^b) - \alpha\chi_{NC}^b - \tilde{G}_{NC} - \tilde{I}_{NC}\right].$$

The introduction of the debt-ceiling rule lowers the ratio of public capital to private capital ($\chi_{DC}^z < \chi_{NC}^z$), raises the ratio of labor income tax revenue to output ($\tilde{T}_{DC}^l > \tilde{T}_{NC}^l$), lowers the ratio of capital income tax revenue to output ($\tilde{T}_{DC}^k(\eta) < \tilde{T}_{NC}^k(\chi_{NC}^b)$), and lowers the ratio of public debt to private capital ($\eta < \chi_{NC}^b$). In our model, the sum of the second and fourth effects dominates the sum of the first and third effects. Thus, the introduction of the debt-ceiling rule raises the economic growth rate along the BGP; that is, $\gamma_{DC} > \gamma_{NC}$.[5]

Lastly, we analyze the effects of the introduction of the debt-ceiling rule in period 0 on each generation's welfare. The welfare of generation $-1$ is represented as

$$V_r^{-1} = \log R(k_0,z_0)(k_0+b_0) + \log[1-T_r^k(k_0,z_0,b_0)] + \theta\log G_r(k_0,z_0,b_0) \quad r = DC, NC.$$

The capital income tax rate and public good provision in period 0 under the debt-ceiling rule are identical to those in the absence of the fiscal rule; that is, $T_{DC}^k(k_0,z_0,b_0) = T_{NC}^k(k_0,z_0,b_0)$ and $G_{DC}(k_0,z_0,b_0) = G_{NC}(k_0,z_0,b_0)$. Thus, the welfare of generation $-1$ under the debt-ceiling rule is also identical to that in the absence of the fiscal rule; that is, $V_{DC}^{-1} = V_{NC}^{-1}$. This implies that the introduction of the debt-ceiling rule has no effect on the welfare of generation $-1$.

The welfare of generation $t$ ($\geq 0$) is represented as

$$\begin{aligned} V_r^t =& (1+\beta)\log w(k_t,z_t) + (1+\beta)\log[1-T_r^k(k_t,z_t,b_t)] + \theta\log G_r(k_t,z_t,b_t) \\ &+ \beta\log R(k_{t+1},z_{t+1}) + \beta\log[1-T_r^k(k_{t+1},z_{t+1},b_{t+1})] + \beta\theta\log G_r(k_{t+1},z_{t+1},b_{t+1}) \quad r = DC, NC, \end{aligned}$$

where irrelevant terms are omitted. Note that the wage rate, labor income tax rates in period $t$, and public good provision in period $t$ are respectively given by

$$w(k_t,z_t) = (1-\alpha)Ak_t^\alpha z_t^{1-\alpha} = (1-\alpha)y_t,$$

$$T_{DC}^l(k_t,z_t,b_t) = \tau_{DC}^l, \quad T_{NC}^l(k_t,z_t,b_t) = \tau_{NC}^l,$$

$$G_{DC}(k_t,z_t,b_t) = G_{NC}(k_t,z_t,b_t) = \frac{\theta[\omega+(1+n)(1-\omega)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}y_t.$$

[5] Comparing $\gamma_{DC}$ of Equation (48) with $\gamma_{NC}$ of Equation (36) gives the result.

Under the debt-ceiling rule, $z_{t+1}/k_{t+1} = \chi^z_{DC}$ and $b_{t+1}/k_{t+1} = \eta$ for $t \geq 0$. Thus, the interest rate and capital income tax rate in period $t+1$ are respectively given by

$$R(k_{t+1}, z_{t+1}) = \alpha A \left(\frac{z_{t+1}}{k_{t+1}}\right)^{1-\alpha} = \alpha A(\chi^z_{DC})^{1-\alpha} \equiv R_{DC},$$

$$T^k_{DC}(k_{t+1}, z_{t+1}, b_{t+1}) = 1 - \frac{\omega}{\alpha(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]}\frac{1}{1+\eta} \equiv \tau^k_{DC}.$$

In the absence of the fiscal rule, $z_{t+1}/k_{t+1} = \chi^z_{NC}$ and $b_{t+1}/k_{t+1} = \chi^b_{NC}$ for $t \geq 0$. This implies that

$$R(k_{t+1}, z_{t+1}) = \alpha A \left(\frac{z_{t+1}}{k_{t+1}}\right)^{1-\alpha} = \alpha A(\chi^z_{NC})^{1-\alpha} \equiv R_{NC},$$

$$T^k_{NC}(k_{t+1}, z_{t+1}, b_{t+1}) = 1 - \frac{\omega}{\alpha(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]}\frac{1}{1+\chi^b_{NC}} \equiv \tau^k_{NC}$$

The property that $\chi^z_{DC} < \chi^z_{NC}$ and $\eta < \chi^b_{NC}$ leads to $R_{DC} < R_{NC}$ and $\tau^k_{DC} < \tau^k_{NC}$; that is, the introduction of the debt-ceiling rule lowers the interest rate and capital income tax rate. Furthermore, the public good provision in period $t+1$ under the debt-ceiling rule and that in the absence of the fiscal rule are respectively given by

$$G_{DC}(k_{t+1}, z_{t+1}, b_{t+1}) = gy_{t+1} = g\gamma_{DC}y_t, \quad G_{NC}(k_{t+1}, z_{t+1}, b_{t+1}) = gy_{t+1} = g\gamma_{NC}y_t.$$

Noting that under the debt-ceiling rule output per capita is given by $y_t = \gamma^t_{DC}y_0$, and that in the absence of the fiscal rule is given by $y_t = \gamma^t_{NC}y_0$. Then, the welfare of generation $t$ can be rewritten as

$$\begin{aligned} V^t_r =& (1+\beta)(1+\theta)\log y_0 + (1+\beta)\log(1-\tau^l_r) \\ &+ \beta \log R_r + \beta\log(1-\tau^k_r) + [\beta\theta + t(1+\beta)(1+\theta)]\log\gamma_r \quad r = DC, NC. \end{aligned}$$

By comparing $V^t_{DC}$ with $V^t_{NC}$, we obtain the following equivalence relation:

$$V^t_{DC} > V^t_{NC} \quad \Leftrightarrow \quad t > \hat{t},$$

where

$$\hat{t} \equiv \frac{1}{(1+\beta)(1+\theta)}\left[\frac{(1+\beta)\log\left(\frac{1-\tau^l_{NC}}{1-\tau^l_{DC}}\right) - \beta\log\left(\frac{1-\tau^k_{DC}}{1-\tau^k_{NC}}\right) + \beta\log\left(\frac{R_{NC}}{R_{DC}}\right)}{\log\left(\frac{\gamma_{DC}}{\gamma_{NC}}\right)} - \beta\theta\right].$$

Thus, the introduction of the debt-ceiling rule lowers the welfare of generation $t < \hat{t}$ while raising the welfare of generation $t > \hat{t}$, implying a trade-off of the fiscal rule between current and future generations. The intuition of this result is as follows. The introduction of the debt-ceiling rule lowers the welfare of generation $t$ by raising the labor income tax rate and lowering the interest rate; meanwhile, it raises the welfare of generation $t$ by raising the capital income tax rate and raising the economic growth rate.

The above discussion can be summarized as Proposition 6.

**Proposition 6.** *In the case where $\eta < \chi^b_{NC}$, the introduction of the debt-ceiling rule raises the economic growth rate along the BGP; that is, $\gamma_{DC} > \gamma_{NC}$. Furthermore, it has the following effects on each generation's welfare.*

1. *It does not affect the welfare of generation $-1$.*

2. *It lowers the welfare of generation $t < \hat{t}$ while raising the welfare of generation $t > \hat{t}$.*

### 4.3 The golden rule

Here, we characterize the politico-economic equilibrium under the golden rule of public finance. We consider the golden rule that the amount of public debt issuance must be less than or equal to the amount of public investment:

$$B_{t+1} \leq I_t \quad \Leftrightarrow \quad b_{t+1} \leq \frac{1}{1+n} i_t.$$

In contrast to Uchida and Ono (2021b), in this study, public investment need not necessarily be financed by issuing public debt (i.e., public investment may be financed entirely by taxation).[6]

Under the simple setup of this study, we can easily derive the politico-economic equilibrium under the golden rule of public finance. From Equations (24) and (25), we obtain

$$B_{NC}(k, z, b) \leq \frac{1}{1+n} I_{NC}(k, z, b).$$

That is, even if any fiscal rule did not exist, the constraint characterizing the golden rule of public finance would not bind. This implies that the politico-economic equilibria in the absence of the fiscal rule and under the golden rule of public finance are identical. Thus, the golden rule of public finance does not affect economic growth patterns and on each generation's welfare.

## 5 Concluding remarks

This study constructs an overlapping generations model in which policies (capital income tax, labor income tax, public good provision, public investment, and public debt) are determined by a probabilistic voting process. We first characterize the Markov perfect politico-economic equilibrium in the absence of any fiscal rule, and derive comparative statics on the size of policies and rate of economic growth. This study focuses on the debt ceiling rule and golden rule of public finance. We show that the introduction of the debt ceiling rule raises the rate of economic growth, and that the debt ceiling rule raises (lowers) the welfare of future generations (the current generations). Regarding the golden rule of public finance, even with no fiscal rule, the constraint which characterizes the golden rule of public finance does not bind. Thus, the golden rule of public finance does not affect the economic growth pattern and each generation's welfare.

Finally, we mention some directions for extension of this study. First, under our simple setup, the golden rule of public finance does not affect economic growth and welfare. However, under a more general setup, this result could change. Second, this study assumes perfectly competitive markets. Introducing market imperfections into the model would bring new insights.

# Appendix

### A.1 Proof of Proposition 1

First, from Equations (6) and (9), the interest rate, $R'$ is represented as

$$R(k', z') = \alpha A (k')^{\alpha-1} (z')^{1-\alpha} = \frac{\alpha A}{(1+n)^{1-\alpha}} (k')^{\alpha-1} i^{1-\alpha},$$

[6]Uchida and Ono (2021b) adopt a golden rule of public finance whereby a certain percentage of public education expenditure is financed by issuing public debt:

$$b_{t+1} = \phi x_t,$$

where $x_t$ is per capita public education expenditure, and $\phi$ is a constant variable which determines the percentage of public education expenditure financed by public debt issuance.

and the political objective function given by Equation (13) is rewritten as

$$
\begin{aligned}
\Omega =& \omega \log R(k,z)(k+b) + (1+n)(1-\omega)(1+\beta)\log w(k,z) \\
&+ \omega \log(1-\tau^k) + (1+n)(1-\omega)(1+\beta)\log(1-\tau^l) \\
&+ \theta[\omega + (1+n)(1-\omega)]\log g + \beta(1-\alpha)(1+n)(1-\omega)\log i \\
&- \beta(1-\alpha)(1+n)(1-\omega)\log k' + \beta(1+n)(1-\omega)\log(1-\tau^{k\prime}) + \beta\theta(1+n)(1-\omega)\log g'.
\end{aligned} \tag{A.1}
$$

Substituting

$$
\begin{aligned}
\tau^{k\prime} &= T^k_{NC}(k',z',b') = 1 - \frac{\delta^k_{NC}}{\alpha}\frac{k'}{k'+b'}, \\
g' &= G_{NC}(k',z',b') = \delta^g_{NC} A (k')^{\alpha}(z')^{1-\alpha}
\end{aligned}
$$

into Equation (A.1) leads to

$$
\begin{aligned}
\Omega =& \omega \log R(k,z)(k+b) + (1+n)(1-\omega)(1+\beta)\log w(k,z) \\
&+ \omega \log(1-\tau^k) + (1+n)(1-\omega)(1+\beta)\log(1-\tau^l) \\
&+ \theta[\omega + (1+n)(1-\omega)]\log g + \beta(1-\alpha)(1+\theta)(1+n)(1-\omega)\log i \\
&+ \alpha\beta(1+\theta)(1+n)(1-\omega)\log k' - \beta(1+n)(1-\omega)\log(k'+b').
\end{aligned} \tag{A.2}
$$

Furthermore, by substituting

$$
\begin{aligned}
k' &= F^k(\tau^k,\tau^l,g,i,k,z,b), \\
k'+b' &= \frac{1}{1+n}\frac{\beta}{1+\beta}(1-\tau^l)w(k,z)
\end{aligned}
$$

into Equation (A.2), we obtain

$$
\begin{aligned}
\Omega =& \omega \log R(k,z)(k+b) + (1+n)(1-\omega)(1+\beta)\log w(k,z) \\
&+ \omega \log(1-\tau^k) + (1+n)(1-\omega)\log(1-\tau^l) \\
&+ \theta[\omega + (1+n)(1-\omega)]\log g + \beta(1-\alpha)(1+\theta)(1+n)(1-\omega)\log i \\
&+ \alpha\beta(1+\theta)(1+n)(1-\omega) \\
&\times \log\left[\frac{\beta}{1+\beta}w(k,z) + \frac{1}{1+\beta}\tau^l w(k,z) + \tau^k R(k,z)(k+b) - R(k,z)b - \frac{2+n}{1+n}g - i\right].
\end{aligned}
$$

The first-order condition with respect to $\tau^k$, $\tau^l$, $g$ and $i$ are given by

$$
\frac{\omega}{1-\tau^k} = \frac{\alpha\beta(1+\theta)(1+n)(1-\omega)R(k,z)(k+b)}{\frac{\beta}{1+\beta}w(k,z) + \frac{1}{1+\beta}\tau^l w(k,z) + \tau^k R(k,z)(k+b) - R(k,z)b - \frac{2+n}{1+n}g - i}, \tag{A.3}
$$

$$
\frac{(1+n)(1-\omega)}{1-\tau^l} = \frac{\alpha\beta(1+\theta)(1+n)(1-\omega)\frac{1}{1+\beta}w(k,z)}{\frac{\beta}{1+\beta}w(k,z) + \frac{1}{1+\beta}\tau^l w(k,z) + \tau^k R(k,z)(k+b) - R(k,z)b - \frac{2+n}{1+n}g - i}, \tag{A.4}
$$

$$
\frac{\theta[\omega+(1+n)(1-\omega)]}{g} = \frac{\alpha\beta(1+\theta)(1+n)(1-\omega)\frac{2+n}{1+n}}{\frac{\beta}{1+\beta}w(k,z) + \frac{1}{1+\beta}\tau^l w(k,z) + \tau^k R(k,z)(k+b) - R(k,z)b - \frac{2+n}{1+n}g - i}, \tag{A.5}
$$

$$
\frac{\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{i} = \frac{\alpha\beta(1+\theta)(1+n)(1-\omega)}{\frac{\beta}{1+\beta}w(k,z) + \frac{1}{1+\beta}\tau^l w(k,z) + \tau^k R(k,z)(k+b) - R(k,z)b - \frac{2+n}{1+n}g - i}. \tag{A.6}
$$

Equations (A.3) - (A.6) correspond to Equations (17) - (20), and these leads to

$$
\tau^k R(k,z)(k+b) = R(k,z)(k+b) - \frac{\omega}{\theta[\omega+(1+n)(1-\omega)]}\frac{2+n}{1+n}g. \tag{A.7}
$$

$$\tau^l w(k,z) = w(k,z) - \frac{(1+\beta)(1+n)(1-\omega)}{\theta[\omega+(1+n)(1-\omega)]}\frac{2+n}{1+n}g, \tag{A.8}$$

$$i = \frac{\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{\theta[\omega+(1+n)(1-\omega)]}\frac{2+n}{1+n}g. \tag{A.9}$$

From Equations (A.7) - (A.9), the denominator of the right-hand side of Equation (A.5) can be rewritten as

$$\begin{aligned} &\frac{\beta}{1+\beta}w(k,z) + \frac{1}{1+\beta}\tau^l w(k,z) + \tau^k R(k,z)(k+b) - R(k,z)b - \frac{2+n}{1+n}g - i \\ &= Ak^\alpha z^{1-\alpha} - \frac{(1+\theta)\{\omega+(1+n)(1-\omega)[1+\beta(1-\alpha)]\}}{\theta[\omega+(1+n)(1-\omega)]}\frac{2+n}{1+n}g. \end{aligned} \tag{A.10}$$

From Equations (A.5) and (A.10), we obtain the public good provision function under NC rule:

$$g = G_{NC}(k,z) \equiv \frac{\theta[\omega+(1+n)(1-\omega)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}\frac{1+n}{2+n}Ak^\alpha z^{1-\alpha}. \tag{A.11}$$

Substituting $g$ of Equation (A.11) into Equations (A.7), (A.8) and (A.9) gives the capital and labor income tax functions and public investment function:

$$\tau^k = T^k_{NC}(k,z,b) \equiv 1 - \frac{\omega}{\alpha(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}\frac{1}{1+b/k}, \tag{A.12}$$

$$\tau^l = T^l_{NC}(k,z,b) \equiv 1 - \frac{(1+\beta)(1+n)(1-\omega)}{(1-\alpha)(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}, \tag{A.13}$$

$$i = I_{NC}(k,z,b) \equiv \frac{\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}Ak^\alpha z^{1-\alpha}. \tag{A.14}$$

Furthermore, substituting $\tau^k$ of Equation (A.12), $\tau^l$ of Equation (A.13), $g$ of Equation (A.11) and $i$ of Equation (A.14) into Equation (12) yields the public debt function:

$$b' = B_{NC}(k,z,b) \equiv \frac{\beta(1-\omega)[1-\alpha(1+\theta)]}{(1+\theta)[\omega+(1+\beta)(1+n)(1-\omega)]}Ak^\alpha z^{1-\alpha}.$$

## A.2 Proof of Proposition 4

First, recall that transition equations of $k$ and $b$ are respectively given by

$$k' = \frac{1}{1+n}\left[\frac{\beta}{1+\beta}w(k,z) + \frac{1}{1+\beta}\tau^l w(k,z) + \tau^k R(k,z)(k+b) - R(k,z)b - \frac{2+n}{1+n}g - i\right], \tag{A.15}$$

$$b' = \frac{1}{1+n}\left[R(k,z)b + \frac{2+n}{1+n}g + i - \tau^l w(k,z) - \tau^k R(k,z)(k+b)\right]. \tag{A.16}$$

Substituting $k'$ of Equation (A.15) and $b'$ of Equation (A.16) into the constraint $b' = \eta k'$ yields

$$\tau^l = \frac{1+\beta}{1+\beta+\eta}\frac{1}{w(k,z)}\left\{(1+\eta)\left[R(k,z)b + \frac{2+n}{1+n}g + i - \tau^k R(k,z)(k+b)\right] - \frac{\beta\eta}{1+\beta}w(k,z)\right\}, \tag{A.17}$$

and substituting $\tau^l$ of Equation (A.17) into Equation (A.15) yields

$$k' = \frac{1}{1+n}\frac{\beta}{1+\beta+\eta}\left[w(k,z) - R(k,z)b - \frac{2+n}{1+n}g - i + \tau^k R(k,z)(k+b)\right]. \tag{A.18}$$

Furthermore, by substituting $\tau^l$ of Equation (A.17) and $k'$ of Equation (A.18) into Equation (A.1), the political objective function is represented as

$$\begin{aligned}\Omega =&(1+\alpha\beta)(1+n)(1-\omega)\log\left[w(k,z)-R(k,z)b-\frac{2+n}{1+n}g-i+\tau^k R(k,z)(k+b)\right]\\&+\omega\log(1-\tau^k)+\theta[\omega+(1+n)(1-\omega)]\log g+\beta(1-\alpha)(1+n)(1-\omega)\log i\\&+\beta(1+n)(1-\omega)\log(1-\tau^{k\prime})+\beta\theta(1+n)(1-\omega)\log g'.\end{aligned}\tag{A.19}$$

We guess the capital income tax rule and the public good provision rule such that

$$[1-T^k_{DC}(k,z,b)]\alpha Ak^{\alpha-1}z^{1-\alpha}(k+b)=\delta^k_{DC}Ak^\alpha z^{1-\alpha}\quad\Leftrightarrow\quad T^k_{DC}(k,z,b)=1-\frac{\delta^k_{DC}}{\alpha}\frac{k}{k+b},\tag{A.20}$$

$$G_{DC}(k,z,b)=\delta^g_{DC}Ak^\alpha z^{1-\alpha}.\tag{A.21}$$

From Equations (A.20) and (A.21) and the constraint $b'=\eta k'$, we obtain

$$1-\tau^{k\prime}=\frac{\delta^k_{DC}}{\alpha}\frac{k'}{k'+b'}=\frac{\delta^k_{DC}}{\alpha}\frac{1}{1+\eta},\tag{A.22}$$

$$g'=\delta^g_{DC}A(k')^\alpha(z')^{1-\alpha}=\delta^g_{DC}\frac{A}{(1+n)^{1-\alpha}}(k')^\alpha i^{1-\alpha}.\tag{A.23}$$

By substituting $1-\tau^{k\prime}$ of Equation (A.22) and $g'$ of Equation (A.23) into Equation (A.19), the political objective function is expressed as

$$\begin{aligned}\Omega =&(1+n)(1-\omega)[1+\alpha\beta(1+\theta)]\log\left[w(k,z)-R(k,z)b-\frac{2+n}{1+n}g-i+\tau^k R(k,z)(k+b)\right]\\&+\omega\log(1-\tau^k)+\theta[\omega+(1+n)(1-\omega)]\log g+\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)\log i.\end{aligned}$$

The first-order conditions with respect to $\tau^k$, $g$ and $i$ are respectively given by

$$\frac{\omega}{1-\tau^k}=\frac{(1+n)(1-\omega)[1+\alpha\beta(1+\theta)]R(k,z)(k+b)}{w(k,z)-R(k,z)b-\frac{2+n}{1+n}g-i+\tau^k R(k,z)(k+b)},\tag{A.24}$$

$$\frac{\theta[\omega+(1+n)(1-\omega)]}{g}=\frac{(1+n)(1-\omega)[1+\alpha\beta(1+\theta)]\frac{2+n}{1+n}}{w(k,z)-R(k,z)b-\frac{2+n}{1+n}g-i+\tau^k R(k,z)(k+b)},\tag{A.25}$$

$$\frac{\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{i}=\frac{(1+n)(1-\omega)[1+\alpha\beta(1+\theta)]}{w(k,z)-R(k,z)b-\frac{2+n}{1+n}g-i+\tau^k R(k,z)(k+b)}.\tag{A.26}$$

Equations (A.24) - (A.26) leads to

$$\tau^k R(k,z)(k+b)=R(k,z)(k+b)-\frac{\omega}{\theta[\omega+(1+n)(1-\omega)]}\frac{2+n}{1+n}g,\tag{A.27}$$

$$i=\frac{\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{\theta[\omega+(1+n)(1-\omega)]}\frac{2+n}{1+n}g.\tag{A.28}$$

From Equations (A.27) and (A.28), the denominator of the right-hand side of Equation (A.25) can be rewritten as

$$\begin{aligned}&w(k,z)-R(k,z)b-\frac{2+n}{1+n}g-i+\tau^k R(k,z)(k+b)\\&=Ak^\alpha z^{1-\alpha}-\frac{\theta[\omega+(1+n)(1-\omega)]+\omega+\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{\theta[\omega+(1+n)(1-\omega)]}\frac{2+n}{1+n}g.\end{aligned}\tag{A.29}$$

From Equations (A.25) and (A.29), we obtain the public good provision function under DC rule:

$$g = G_{DC}(k,z,b) \equiv \frac{\theta[\omega + (1+n)(1-\omega)]}{(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]} \frac{1+n}{2+n} Ak^{\alpha} z^{1-\alpha}. \tag{A.30}$$

By substituting $g$ of Equation (A.30) into Equations (A.27) and (A.28), we obtain the capital income tax function and public investment function:

$$\tau^k = T^k_{DC}(k,z,b) \equiv 1 - \frac{\omega}{\alpha(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]} \frac{1}{1+b/k}, \tag{A.31}$$

$$i = I_{DC}(k,z,b) \equiv \frac{\beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]} Ak^{\alpha} z^{1-\alpha}. \tag{A.32}$$

Furthermore, substituting $g$ of Equation (A.30), $\tau^k$ of Equation (A.31) and $i$ of Equation (A.32) into Equation (A.17) gives the labor income tax function:

$$\begin{aligned} \tau^l &= T^l_{DC}(k,z,b) \\ &\equiv \frac{(1+\beta)(1+\eta)}{(1-\alpha)1+\beta+\eta} \left\{ \frac{\theta[\omega + (1+n)(1-\omega)] + \omega + \beta(1-\alpha)(1+\theta)(1+n)(1-\omega)}{(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]} - \alpha \right\} \\ &\quad - \frac{1+\beta}{1+\beta+\eta} \frac{\beta\eta}{1+\beta}. \end{aligned}$$

Lastly, we derive the public debt function. Substituting $g$ of Equation (A.30), $\tau^k$ of Equation (A.31) and $i$ of Equation (A.32) into Equation (A.18) leads to

$$k' = \frac{\beta}{1+\beta+\eta} \frac{(1-\omega)[1+\alpha\beta(1+\theta)]}{(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]} Ak^{\alpha} z^{1-\alpha}. \tag{A.33}$$

From (A.33) and the constraint $b' = \eta k'$, we obtain the public debt function:

$$b' = B_{DC}(k,z,b) \equiv \frac{\beta\eta}{1+\beta+\eta} \frac{(1-\omega)[1+\alpha\beta(1+\theta)]}{(1+\theta)[\omega + (1+\beta)(1+n)(1-\omega)]} Ak^{\alpha} z^{1-\alpha}.$$

## A.3 Proof of Lemma 2

Differentiating $\tau_{DC}$ with respect to $\eta$, it is shown that $\tau_{DC}$ is decreasing with respect to $\eta$:

$$\frac{\partial \tau_{DC}}{\partial \eta} = -\frac{\beta(1+\beta)(1+n)(1-\omega)[1+\alpha\beta(1+\theta)]}{(1-\alpha)(1+\theta)(1+\beta+\eta)^2[\omega + (1+\beta)(1+n)(1-\omega)]} < 0.$$

Furthermore, a simple calculation shows that $\tau_{DC} = \tau_{NC}$ when $\eta = \chi^b_{NC}$. Thus, $\tau_{DC} > \tau_{NC}$ if $\eta < \chi^b_{NC}$.

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