

# Recent Topics on Nonlinear Analysis

**Period :** August 5 (Sat.), 2017, 13:30–17:50

**Place :** Big Seminar Room (E408), Department of Mathematics, Osaka City University

## —Program—

**13:30 – 14:20: Cheikh Birahim, Ndiaye** (Universität Basel)

**Algebraic topological methods and uniformization in conformal geometry:**

Abstract: In this talk we plan to describe joint works with Martin Mayer from Centro De Giorgi in Pisa on some uniformization theorems in conformal geometry, namely the boundary Yamabe problem, the Riemann mapping problem of Cherrier-Escobar and the Fractional Yamabe problem of Gonzalez-Qing. We will show how the Barycenter technique of Bahri-Coron sometimes called algebraic topological argument naturally applies to those local and non-local Yamabe type problems focusing on its link with the quantization and strong interaction phenomena satisfied by those variational problems.

**14:40 – 15:05: Megumi, Sano** (Osaka City University, D3)

**Sharp critical Rellich inequality in four dimension:**

Abstract: It is well-known that Rellich inequalities hold true on higher order Sobolev spaces  $W^{2,p}(B_1)$  in the subcritical case  $p < N/2$ . Their optimal constants and the non-attainability of those are also known. In this talk, we consider the Rellich type inequality on the critical Sobolev space  $W^{2,2}(B_1)$  in four dimension. Note that the potential function of our inequality has singularities at the origin and at the boundary. This is a joint work with Prof. F. Takahashi (Osaka City Univ.).

**15:15 – 15:40: Masato, Hashizume** (Osaka City University, D3)

**Semilinear elliptic equation involving a variable exponent:**

Abstract: We consider the existence of positive solutions of a semilinear elliptic equation involving a variable exponent. By assumption of the variable exponent, the equation becomes linear at one point. We study the relation between the existence of a solution and the behavior of the variable exponent at the point. This is a joint work with Megumi Sano (OCU).

**15:50 – 16:40: Daisuke, Naimen** (Muroran Institute of Technology)

**Blow-up analysis for nodal radial solutions in Moser-Trudinger critical equations in  $\mathbb{R}^2$ :**

Abstract: We consider low energy nodal radial solutions of Moser-Trudinger critical equations in  $\mathbb{R}^2$ . We study the asymptotic behaviour of them as the growth rate of the nonlinearity goes to a borderline between the existence and nonexistence of nodal radial solutions. We show if the solution has  $k$ -interior zeros, it exhibits a multiple blow-up behaviour on the first  $k$  nodal sets while, on the  $(k + 1)$ -th one, it strongly converges to the least energy solution of a critical equation of the borderline case. We also prove that each concentration part, with an appropriate scaling, converges to a solution of the classical Liouville problem in  $\mathbb{R}^2$ . This talk is based on a joint work with M. Grossi at Sapienza University of Rome.

**17:00 – 17:50: Cheikh Birahim, Ndiaye** (Universität Basel)

**Vanishing viscosity and topological methods for the Q-curvature equation:**

Abstract: In this talk, we plan to present a variational theory-based partly on a joint work with Mohameden Ahmedou from the University of Giessen in Germany for the resonant Q-curvature equation on closed 4-dimensional Riemannian manifolds. Precisely, under a nondegeneracy condition, we will show how the refined bubbling analysis of some natural vanishing viscosity solutions leads to the characterization of the singularities of the associated variational problem via the establishment of a Morse Lemma around them, and the compactness of the equation via a Chen-Lin exact bubbling rate formula. Moreover, we will show how the latter Chen-Lin exact bubbling rate formula leads naturally to existence results under a Positive Mass type assumption, and that how when combined with the Morse Lemma at infinity—namely the one around the singularities of the variational problem—they provide a formula for the Leray-Schauder degree of the resonant Q-curvature equation, and hence providing Bahri-Coron and Chang-Yang Poincaré-Hopf index type formula for existence. We will show also how the strength of the Morse Lemma at infinity gives K. C. Chang and Malchiodi-Struwe system type existence results. Finally, we will explain how to use the Morse Lemma at infinity and Conley theory to develop algebraic topological arguments for existence à la Bahri-Coron, even if the Paneitz functional does not verify the quantization property like the Yamabe functional.