Future research plans
(April, 2025)

Osaka Metropolitan University
Tatsuya Hosono

The citation numbers stated below align with the article numbers in the publication list.

1. Boundedness and asymptotic behavior for chemotaxis systems: From previous researches on critical mass phenomena (mainly [3, 4, 6]), we identified the threshold M^* of initial mass that determines global solvability for the chemotaxis system in n space dimensions. We show that solutions with initial mass less than M^* remain uniformly bounded in time. In whole space, it is difficult to derive time-independent estimates for solutions due to the fact that we are required to control the behavior of solutions as $|x| \to \infty$. We temporarily focus on radial symmetric solutions and their relationship with stationary solutions. Using rearrangement arguments,we then analyze boundedness for general solutions, including L^p-decay estimates and asymptotic behavior of global bounded solutions.

2. Large time behavior for chemotaxis systems with nonlinear diffusion: We here consider the chemotaxis system with nonlinear diffusion, given by $\partial_t u - \nabla \cdot (u^{\alpha-1}\nabla u)$ ($\alpha \in \mathbb{R}$). In n-dimensional space, the balance between diffusion and nonlinear interaction terms determines the critical diffusion exponent; $\alpha_{c,n} := 2 - n/4$. The behavior of solutions is expected to change at this threshold. The work, with Ph. Laurençot (Université Savoie Mont Blanc), already shows critical mass phenomena for linear diffusion $n = 4$ ([4]) and degenerate diffusion $n \geq 5$. In lower dimensions $n \leq 3$, the behavior of the solution deviates from predictions due to fast diffusion structures, and traditional methods to fail due to the strength of nonlinearity. Thus, we focus on new methods to construct solutions, analyze their properties, and specifically explore how the chemotaxis term affects concentration and large time behavior of solutions, especially around the critical exponent.

3. Analysis via energy functional for nonlinear diffusion equations and its applications: We aim to derive a monotonicity estimate for an energy functional related to the Fisher information in chemotaxis systems. These estimates are useful for understanding regularity, global existence, and blowup behavior of solutions. In joint work with Tomasz Cieślak (Polish Academy of Sciences) and Kentaro Fujie (Tohoku University), we are working on a time monotonicity estimate, focusing on the convexity of the second derivative of the energy for the generalized nonlinear diffusion equation $\partial_t u = \nabla \cdot (a(u)\nabla u)$ $(a(0) < +\infty)$. Our goal is to establish the convergence of solutions to steady states, determine their rate, and further refine solution estimates, particularly for chemotaxis systems.

4. Analysis of sign-changing solutions to nonlinear diffusion equations: We study sign-changing solutions to nonlinear diffusion equations $\partial_t u = \Delta(|u|^{m-1}u)$. While the construction of nonnegative solutions has been widely explored since Aronson–Peletier (1981), sign-changing solutions are more challenging. A recent result by Brasco–Volzone (2022) investigates initial-boundary value problems for porous medium equations without restrictions on sign-changing initial data. In this study, we focus on the elliptic variational structure and extend the analysis to initial value problems in the entire space and fast diffusion-type equations. Additionally, we examine cases where a reaction term $f(u)$ is added, using comparison principles and other methods to analyze the compact support and finite propagation of sign-changing solutions, as seen in the Brasco–Volzone result.