Research Summary
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Research Summary

I have studied the large time behavior of solutions to partial differential equations (PDEs) derived from chemotaxis phenomena, known as chemotaxis systems. These systems are typically modeled as parabolic-elliptic or fully parabolic systems and describe the migration of cells or organisms driven by chemical signals. The dynamics are governed by both "diffusion" and "transport effects" arising from the nonlinear interaction in chemical reactions, which can be observed in various biochemical contexts.

My research focuses on the mass conservation law (L^1-conservation) and the entropy structure of chemotaxis systems, as well as the balance between opposing effects such as diffusion and concentration. I am particularly interested in how the solution changes depending on the mass's magnitude and shape of the initial data.

Large time behavior of solutions and critical mass phenomena ([2, 3, 4, 6])**:** One key topics in the study of such chemotaxis systems is the critical mass phenomenon, where a threshold value M^* for the initial mass determines whether finite-time blowup of solutions occurs. Specifically, the solution with the initial mass below M^* exists globally in time and remains stable. However if the initial mass exceeds M^*, the solution may blow up in finite time. In fact, it is known that for the two-dimensional whole space, $M^* = 8\pi$.

I have so far considered the large time behavior of solutions for the Cauchy problem of a chemotaxis system related to Alzheimer's disease, and have identified the four dimensional critical mass phenomena with a threshold value of $(8\pi)^2$. The study of the problem on the whole space requires a different approach compared to that used in a bounded domain by Fujie–Senba (2017, 2019), in particular to control the behavior of the solutions as $|x| \to \infty$. We introduce the modified entropy, the Brézis–Merle type inequality for a 4th order elliptic equation and a new type functional inequality using rearrangement arguments, so that a priori estimates for solutions are derived in [2, 3, 4]. In [6], a generalization of a parabolic-elliptic chemotaxis system is studied and the n dimensional threshold value and shape of initial data for the global solvability are investigated.

Large time behavior of solutions in super critical regions ([1, 5])**:** In scaling super critical regions, identifying the threshold becomes more difficult and remains open. However, by using Shannon's inequality and its optimal constant, we derived an estimate under the entropy, and obtained initial conditions for finite-time blowup of solutions. Furthermore, by imposing a smallness condition on the initial data, we also established the global well-posedness for the problem.