Future research

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(1) On the relation between 2-Calabi-Yau triangulated categories and generalized q-deformed rational numbers.

This study considers the $(n+1)$-braid group action on the 2-Calabi-Yau triangulated category $\mathcal{C}_n$ and aims to clarify the combinatorial structure of spherical objects on $\mathcal{C}_n$. Specifically, we construct generalized q-deformed rational numbers (q-deformation of rational points in an $(n-1)$-dimensional projective space). I will clarify the relation between the $(n+1)$-braid group action on $\mathcal{C}_n$ and the $PSL_q(n,Z)$-action on the set of generalized q-deformed rational numbers and extend our previous results on q-deformed rational numbers to generalized q-deformed rational numbers.

(2) Arithmetic decomposition problem for q-deformed rational numbers

It is conjectured that if the denominator of an irreducible fractions is prime, the denominator polynomial of its q-deformed rational number is irreducible in $Z[q]$, and moreover, the same holds for the left q-deformed rational numbers. This has been checked by computer for primes below 730, but in general it remains open. As an example for the case left q-deformed rational numbers, for a prime p, $q^p + q^{p-2} + \cdots + q + 1$ is expected to be irreducible in $Z[q]$, but even such a special case is open. We would like to continue working on this problem.

(3) The generalized Specht ideal and its Gröbner basis

A universal Gröbner basis of an ideal of the polynomial ring $K[x_1, \dots, x_n]$ over a field K is that it is a Gröbner basis for any term order. It is easy to see that the Gröbner basis of the Specht ideals I_μ constructed in the previous work is universal. On the other hand, the Gröbner basis of $I_{\mu,k}$ that we constructed is also expected to be universal. Still, even the partial result we obtained requires a very complicated proof. Because the symmetry of the corresponding variable x_1 is lost by putting k copies of 1 in the tableau. We want to work on this problem as well.

For a further generalization of $I_{\mu,k}$, we take j positive integers $k_1, \dots, k_j$, and a partition μ of $m := k_1 + \cdots + k_j$, and consider the tableaux T of shape μ in which each j appears in k_j times (note that such tableaux are treated in "Kostka's theorem"). Now we have the reduced Specht polynomial of T and the ideal $\mathrm{I}_{\mu,k_1,\dots,k_j}$ generated by all of these polynomials. We want to construct a Gröbner basis of this ideal and show that the ideal is radical.