Research summary

REN XIN

Research 1: On q-deformed real numbers and q-deformed Hurwitz's theorem in number theory

For a parameter q, the q-deformed rational numbers were introduced by S. Morier-Genoud and V. Ovsienko by considering continued fractions and q-deformed modular groups. This new notion has recently been used to study objects in many areas of mathematics. Moreover, as an extension, they also introduced the notion of q-deformed real numbers (as power series in q) by using rational approximations of real numbers. Let q be a complex number. In the context of q-deformed real numbers, Hurwitz's theorem in number theory should be translated to the following conjecture. "Among the radiuses of convergence of q-deformed real numbers, the case golden ratio $(1+\sqrt{5})/2$ is the smallest." I proved this conjecture in several cases involving metallic numbers (i.e., quadratic irrational numbers expressed as $n+\sqrt{n^2+4}/2$), and for each term in the rational sequence converging to metallic numbers, gave a lower bound on the radius of convergence of their q-deformed real numbers.

Research 2: Gröbner basis and generalized Specht ideal

Shuo Yen Robert Li and Wen Ching Winnie Li introduced a class of ideals in the polynomial ring $K[x_1, \dots, x_n]$ to connect commutative algebra with the problem of computing maximal independent sets of graphs. Here we call their ideals Li-Li ideals. We studied a class of ideals which generalizes both Specht ideals and radical Li-Li ideals. Specifically, for a fixed positive integer k, we prepared a partition μ of $n+k-1$, and defined the ideal $I_{\mu,k}$ as the one generated by the generalized Specht polynomials of the Young tableau given by putting $2, \dots, n$ and k copies of 1 to the $n+k-1$ boxes of the Young diagram of shape μ. We proved that$I_{\mu,k}$ is a radical ideal. Furthermore, we constructed a Gröbner basis for this ideal.

Research 3: 2-Calabi–Yau triangulated categories and q-deformed rational numbers

The 2-Calabi-Yau triangulated categories given by the zigzag algebras of quivers were introduced by Khovanov and Seidel. Bapat, Becker and Licata proved that the combinatorial structure of spherical objects in the 2-Calabi-Yau triangulated category $\mathcal{C}_2$ given by the quiver of type A_2 can be expressed using q-deformed rational numbers and left q-deformed rational numbers (which is another q-deformation, and the counter part of the original one).

Considering negative continued fractions, I obtained the Farey sum formula for left q-deformed rational numbers. Using this formula, I gave a combinatorial proof that the normalized Jones polynomial of rational links can be computed using only the numerators of left q-deformed rational numbers. Furthermore, for the Farey sum formula for rational numbers, I showed an analogous formula for spherical objects on $\mathcal{C}_2$. In connection with the quantization of the rational approximation, we also clarified the relation between the q-deformed real generating function and the spherical object on $\mathcal{C}_2$ for the quadratic irrational numbers.

Research 4: Arithmetic properties concerning q-deformed rational numbers

We study some arithmetic properties concerning q-deformed rational numbers. For two rational numbers with the same denominator, we gave sufficient conditions for their denominator polynomials to be equal and necessary and sufficient conditions for the denominator polynomial to be palindromic. Furthermore, some properties were obtained for the factorization of the numerator and denominator polynomials.