Toshiya Yurikusa

Future Research Plan

Building upon my previous work, I plan to pursue the following research directions.

1. Complete classification of cluster algebras with dense g-vector fans

Although I have conducted a *partial classification* of cluster algebras that possess dense g-vector fans, the classification of exceptional types (such as X_6, $F_4^{(*,+)}$, etc.) remains unresolved. I aim to achieve a complete classification by analyzing these exceptional types. In particular, by using *extended affine root systems* [S85], I intend to extend the known relationship between g-vector fans of affine type and *Cambrian fans* [RS18], and thereby analyze the g-vector fans of exceptional types in detail. As a consequence of this study, I aim to demonstrate the denseness of the corresponding g-vector fans.

2. Toward a unified understanding of tameness in τ-tilting theory

In τ-tilting theory, the notion of τ-tilting finiteness, which is analogous to finite representation type, is characterized by the completeness of the g-vector fan [DIJ19]. Based on this property and Theorem 1, I introduced a class of algebras whose g-vector fans are dense as an analogue of tame type—these are called *g-tame algebras*. In addition, several alternative notions analogous to tameness in τ-tilting theory have been proposed, such as τ-tilting tameness and E-tameness. My goal is to construct a *unified framework for understanding tameness* in the context of τ-tilting theory.

As a further direction, I aim to deepen the understanding of τ-tilting theory and the structure of derived categories, and to investigate geometric and combinatorial descriptions of g-vector structures in more general representation types. In particular, I plan to study the behavior of mutations in the cluster-theoretic framework and to track the changes in the associated sets of g-vectors, with the goal of elucidating general properties such as their connectivity, denseness, and geometric shape.

In addition, based on recent developments relating τ-tilting theory and semi-invariants, I plan to focus on the properties of wall and chamber structures determined by stability conditions, and to investigate their relationships with toric structures and hyperplane arrangements. This project aims to further integrate spatial structures in representation theory with combinatorics.

Through these research directions, I intend to continue developing a research program that originates from τ-tilting theory and extends across multiple fields, including derived categories, stability conditions, topology, and combinatorics.

Reference

[DIJ19] L. Demonet, O. Iyama, and G. Jasso. *τ-tilting finite algebras, bricks, and g-vectors*. International Mathematics Research Notices, 2019(3):852–892, 2019.

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[S85] K. Saito. Extended affine root systems I (coxeter transformations). Publications of the Research Institute for Mathematical Sciences, 21(1):75–179, 1985.