Toshiya Yurikusa

(2) Summary of Research Achievements and Future Research Plan

Summary of Research Achievements

My research has primarily focused on cluster algebra theory and the representation theory of finite-dimensional algebras, aiming to integrate these two fields. In particular, I have investigated the combinatorial properties of cluster algebras arising from surfaces and the denseness of the g-vector fan, and have published several representative papers on these topics. These studies aim to clarify the deep connections between cluster algebras and τ-tilting theory, and to offer a new framework within mathematics. In what follows, I focus on the latter line of research.

Cluster algebras are commutative algebras equipped with a combinatorial structure called mutation. The g-vector, which lies at the core of this theory, provides a numerical invariant associated with cluster variables, the fundamental constituents of cluster algebras. The *g-vector fan* formed by these g-vectors offers a geometric representation of the underlying algebraic structure. It not only provides insights into the construction of bases in cluster algebras, but also plays a crucial role in addressing classical conjectures such as the positivity conjecture and the sign-coherence conjecture.

On the other hand, in the representation theory of finite-dimensional algebras, *τ-tilting theory* extends classical tilting theory and plays a central role in the analysis of module categories. In this theory, the g-vector fan serves as a geometric structure that describes the properties of important objects such as stability spaces and torsion pairs in module categories. Moreover, when a cluster algebra admits a categorification by a finite-dimensional algebra, its g-vector fan is known to appear as a subset of the g-vector fan of the algebra.

I have focused on the property of the *denseness* of the g-vector fan and developed my research around this notion. The first result in this line of work was to prove, using geometric methods, that algebras associated with surfaces have dense g-vector fans [Y20]. By rephrasing this method in categorical terms, I extended the approach to apply to classical finite-dimensional algebras. These algebras are known to be classified into two types: **tame** and wild, with important examples such as string algebras and Brauer graph algebras belonging to the tame type. Applying the above method to this classical class, I obtained the following result:

Theorem 1 ([PY23]). Every tame algebra has a dense g-vector fan.

This result provides a new perspective for the classification and structural understanding of finite-dimensional algebras. Furthermore, the study of the denseness of g-vector fans has led to the following applications:

- **Connectivity of exchange graphs:** In τ-tilting theory, determining whether the exchange graph formed by τ-tilting modules is connected is a fundamental question.

Using the denseness of the g-vector fan, I *proved the connectivity of the exchange graph for algebras associated with surfaces* [Y20].

- **Scattering diagrams:** For classes where the g-vector fan is dense, I analyzed the differences between scattering diagrams in cluster algebras and those in finite-dimensional algebras via the notion of denseness [PY23].

These results promote a unified understanding of geometric and algebraic structures.

Reference

[PY23] P. Plamondon and T. Yurikusa. *Tame algebras have dense g-vector fans.* International Mathematics Research Notices, 2023(4):2701–2747, 2023. with an appendix by B. Keller.

[Y20] T. Yurikusa. *Density of g-vector cones from triangulated surfaces.* International Mathematics Research Notices, 2020(21):8081–8119, 2020.