Research Plan

Masahide Iwakiri

I am interested in surfaces in the 4-dimensional space. Until now, we have often researched embedded surfaces in the 4-dimensional space (surface-links) in this research area. I also have studied them in the papers [2-5,7] in the list of publications. I think generic immersed surfaces in the 4-dimensional space (singular surface-links) are worth researching, but they have rarely researched. I have studied them via singular surface braids (cf.[1,6,9]). In these papers, I researched them with respect to 1-handle surgeries and crossing changes. Singular surface braids are represented by graphs in a 2-disk, which are called charts. I would like to study in the following themes:

- Singular ribbon surface-links
- Unknotting numbers associated with 1-handle surgeries and crossing changes
- Determination of singular surface-links with small w-indices

• Singular ribbon surface-links

We call a singular surface-link obtained from some trivial singular surface-knots by a finite number of 1-handle surgeries and crossing changes a *singular ribbon surface-link*. Recently, in [9], we proved that the w-index of a singular surface-link F is 0 if and only if F is a singular ribbon surface-link, which was known for surface-links. We also proved that if F is represented by a $-$amphichiral chart, then F is a singular ribbon surface-link. I would like to consider when the converse is true. In a related matter, we also consider a normal form of singular ribbon surface-links. For ribbon surface-links, it is known that it is represented by some 2-disks and bands in the 3-dimensional space. We also consider similar representation of them. I am also interested in the relationship between that the minimal triple point number of a singular spherical-knot F is 0 and that F is a singular ribbon spherical-knot, which are same for spherical-knots.

• Unknotting numbers associated with 1-handle surgeries and crossing changes

It is known that 1-handle surgeries and crossing changes are unknotting operations, respectively. I would like to study unknotting numbers associated with them, respectively. In [9], we proved that unknotting numbers associated with them of a non-trivial singular surface-link with braid index 3 are 1. In the proof, it is a key point that it is a singular ribbon surface-link. Similarly, I also consider unknotting numbers for singular ribbon surface-links. To determine their unknotting numbers, I will study charts of them. I also consider the fundamental groups of them, the Alexander invariant and coloring numbers to give the lower bound of unknotting numbers.

• Determination of singular surface-links with small w-indices

We know several results with respect to the w-indices of surface-links. It is known that the w-index of a surface-link F is 0 if and only if F is ribbon, and there are no surface-links with w-index $1, 2, 3$ or 5. In [7], we gave lower bounds of w-indices via quandle cocycle invariants. As stated above, we proved that the w-index of a singular surface-link F is 0 if and only if F is a singular ribbon surface-link. Here, I would like to determine the minimal number that is the w-index of a non-singular ribbon surface-link, and characterize them. By invariants, I would like to give lower bounds of w-indices of singular surface-links.