

Research Result

- (1) Toward understanding on GKM-theory from Morse theoretic point of view, I studied invariant Morse-Smale functions on compact manifolds in view of intersection of stable and unstable manifolds. In this setting, it is often the case that there exists no pair of critical points whose Morse indices differ by 1. Accordingly, I investigated the structure of intersections corresponding to pairs of critical points whose Morse indices differ by 2, and proved that each connected component is equivariantly diffeomorphic to the 2-sphere with the standard circle action. Since the fixed point set and the critical point set are the same things in this situation, we can observe that a situation similar to that of GKM-theory appears.
- (2) I studied mixed link which is a variant of the usual link. I defined the Alexander polynomial of a mixed link and proved a relation between the Alexander polynomial of the usual link obtained by resolving the twisted part.
- (3) Let us consider a smooth action of a compact Lie group on a compact manifold, whose fixed point set is finite. We call an open covering indexed by the fixed point set representation covering if each open set is invariant under the group action and is equivariantly diffeomorphic to the tangential representation at the corresponding fixed point.

In this research, I proved that if there exists an equivariant hyperbolic diffeomorphism satisfying a certain convergence condition, there also exists a representation covering on the manifold. As a corollary, it follows that the existence of an invariant Morse function implies the existence of a representation covering of the manifold. Conversely, one finds that the existence of a representation covering gives an obstruction for the existence of an invariant Morse function. In addition, as a converse of the above result, I proved that the existence of a certain representation covering implies the existence of a certain equivariant hyperbolic diffeomorphism when the Lie group is the compact torus and the manifold is a complex manifold.

Moreover, by using this result, it turns out that there are infinitely many torus manifolds having no invariant Morse functions. This is in contrast to the well-known existence theorem of Morse functions and the existence theorem of Wasserman concerning a certain invariant Bott-Morse functions.