Research Plan

Building upon the results we obtained, we will consider the following problems:

1. Construction of semi-discrete CMC surfaces in Riemannian spaceforms

Research on semi-discrete CMC surfaces in Euclidean 3-space had been started by Müller, but he primarily focused on their theoretical aspects, so gave few examples of semi-discrete CMC surfaces. Recently, Wolfgang Carl (Technical University of Graz) derived a matrix representation for semi-discrete non-zero CMC surfaces in Euclidean 3-space, and showed that any semi-discrete CMC surfaces can be described by solving a pair of matrix-valued differential and difference equations. With Carl, applying matrix factorizing theorems, we will give a new construction of semi-discrete CMC surfaces in Euclidean 3-space. After we obtain the construction, we will extend the result to semi-discrete CMC surfaces in 3-dimensional Riemannian spaceforms. Moreover, we will construct examples and analyze singularities of semi-discrete constant positive Gaussian curvature surfaces obtained by taking parallel surfaces of semi-discrete CMC surfaces.

2. Construction of discrete CMC surfaces in Lorentzian spaceforms

As an extension of the previous works in [9], [12], we will describe discrete CMC surfaces in 3-dimensional Lorentian spaceforms. Unlike the case in Euclidean 3-space, and like the case in Minkowski 3-space, because of the behavior of smooth surfaces, we can expect that discrete CMC surfaces in 3-dimensional Lorentzian spaceforms may have singularities. First we will determine a suitable pair of matrix-valued difference equations and derive an associated discrete integrable equation. Moreover, we will derive a new construction of discrete CMC surfaces in Lorentian spaceforms by applying the matrix factorizing theorems, and analyze their singularities using both the previous method in [2] and the analysis of the corresponding discrete sinh-Gordon equation. If time permits, we will also try to describe a semi-discrete analogue.

3. Research on discrete timelike isothermic surfaces

First we will complete a theory of discrete timelike isothermic surfaces in 3-dimensional spaceforms. In a paper [13] in preparation, we are establishing a new theory of discrete timelike isothermic surfaces in Minkowski 3-space, and the result itself was already briefly introduced in [7]. We already obtained a Weierstrass-type representation for discrete timelike minimal surfaces, so we will analyze their singularities. Moreover, by extending this result, we will establish a general theory of discrete timelike isothermic surfaces. As an application, we will derive a Weierstrass-type representation for discrete timelike CMC 1 surfaces in AdS 3-space and analyze their singularities.

4. Analyzing singularities of general discretized surfaces

Singularities of discrete surfaces had already been defined in [2], [5], but, except for the case of discrete linear Weingarten surfaces with Weierstrass-type representations, it is much harder to analyze singularities of general discrete surfaces. The reason is that we cannot differentiate in the realm of discrete differential geometry. Furthermore, singularities of semi-discrete surfaces had been considered only in [4], [6], [8], [10], so we have not yet defined singularities of general semi-discrete surfaces. It is hard to analyze singularities of general discretized surfaces, so we will first start by considering discretized constant Gaussian curvature surfaces, which do not have Weierstrass-type representations. Discretized constant positive Gaussian curvature surfaces can be obtained by taking parallel surfaces of discretized CMC surfaces, and discrete constant negative Gaussian curvature surfaces were already given by Schief. After we clarify singularities of discretized constant Gaussian curvature surfaces, we will characterize singularities of general discretized surfaces.