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1 Research plan

1.1 Classification of blow-up solutions

I will show the existence of particular type II blow-up solutions and their quantitative information for some nonlinear parabolic equations in order to classify all the possible blow-up solutions. As for the Fujita equation, I have constructed some type II blow-up solutions whose mechanisms are different from ones in the literature [1,2]. Due to this, a major open problem on classification of radial blow-up solutions was solved. The next goal in this direction is:

- Establishing refined asymptotics near singularity of general radial blow-up solutions for the Fujita equation
- Extending the classification result to cover the range of the Joseph-Lundgren nonlinerity
- Application of the technique and results developed for the Fujita equation to a number of related nonlinear problems.

This last issue includes, in particular, harmonic map heat flow and Keller–Segel chemotaxis model. Related results are expected, though we expect onset of a number of technical difficulties due to the strong nonlinearity of those equations. I have already obtained some results that support this idea. In particular, I have already obtained a companion paper [10] on harmonic map heat flow as a continuation of the previous work [3]. The paper [10] has been accepted for publication in the journal "*Nonlinearity*", which is a prestigious journal on nonlinear mathematics. The next goal is to classify all the radial blow-up solutions in terms of blow-up rate. After obtaining the expected classification results, I suspect that each model displays different phenomena in, for instance, generic blow-up patterns or stability properties of blow-up mechanisms.

1.2 Other topics

The study of the process of melting ice body, so called **Stefan problem**, is a classical topic in applied analysis. The simplest setting is the situation where an ice body surrounded by water is melting. The temperature of water changes according to the classical heat equation and the boundary between ice and water is expressed by a function of time-variable. The function describing the boundary, as well as a function describing the temperature, is unknown. This type of problem is called, in general, **free boundary problem**. Among the free boundary problems, the Stefan problem has been studied by various approaches such as PDE or functional analytic methods and numerical ones. I would like to study this problem by using the approach that I have developed in the previous research to develop the mathematical theory of the Stefan problem.

Another research direction is concerning the aggregation phenomena in a chemotaxis model. In my previous study [4] with Y. Sugiyama and J. J. L. Velázquez, a specific blow-up solution was constructed in the formal level by meas of matched asymptotic expansions. This solution shows the aggregation of multiple peaks at the blow-up time and produces the Dirac mass whose strength is a multiple of 8π. Such unusual aggregation was used to be considered to be absent until this work. Since this sought-for solution has to be non-radial, the proof of its actual existence is far from obvious. Before dealing with this problem, I will try to obtain analogous results for rather simpler (but related) problems, such as the Fujita equation with critical Sobolev exponent by extending the techniques having been developed in [1,2,10].