RESEARCH RESULTS

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Riemannian symmetric spaces are fundamental and important object in differential geometry. We investigate totally geodesic submanifolds in Riemannian symmetric spaces which are also Riemannian symmetric spaces. It is a fundamental problem to classify the totally geodesic submanifolds in Riemannian symmetric spaces. Many results concerning this problem have been obtained but we have not obtained complete answer yet. The classification of the totally geodesic submanifolds in compact Riemannian symmetric spaces of rank one obtained by J. A. Wolf in 1963. B. Y. Chen and T. Nagano gave the local classification of the totally geodesic submanifolds in compact connected irreducible symmetric spaces of rank two in 1978. By using Chen–Nagano's method, which makes use of the theory of polars and meridians, we give the list of all maximal totally geodesic submanifolds in compact connected irreducible symmetric spaces of rank two in [1].

Totally geodesic submanifolds in Riemannian manifolds are minimal submanifolds. This fact gives us the problem to determine the stability of a totally geodesic submanifold as a minimal submanifold. We determined the stability of maximal totally geodesic submanifolds in compact connected irreducible Riemannian symmetric spaces of rank two which classified in [1] (in [2]).

An action of a Lie group on a Riemannian manifold is a cohomogeneity one action if the codimension of a principal orbit is one. A. Kollross classified all of the cohomogeneity one action on irreducible Riemannian symmetric spaces of compact type. In general, a singular orbit of a cohomogeneity one action is not a totally geodesic. When a singular orbit is a reflective submanifold, namely the orbit is a totally geodesic, it gives us the problem to determine the stability. In this problem, we proved that certain reflective submanifolds of irreducible Riemannian symmetric space of compact type which are singular orbits are stable. Here there is no restriction on the rank of Riemannian symmetric spaces ([3]).