Research results

Toshiyuki MIYAUCHI

I study self-homotopy and Lusternik-Schnirelmann category.

In the homotopy theory, a fundamental problem is to determine the group structure of the homotopy set of sphere mappings $[S^m, S^n] = \pi_m(S^n)$. Many results on $\pi_m(S^n)$ are obtained by many authors, e.g, H. Toda, M. Mimura, M. Mahowald, N. Oda and etc. The real n-projective space P^n is one of the most important spaces other than a sphere. To determine the group structure of $[\Sigma^k \mathrm{P}^m, \Sigma^\ell \mathrm{P}^n]$ is interesting and one of my final subjects. As the first step,I study the group structure of the self-homotopy sets $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$ corresponding to $\pi_k(S^k) \cong \mathbb{Z}$. Earlier, the group structure of $[\Sigma \mathrm{P}^n, \Sigma \mathrm{P}^n]$ for $n \leq 4$ was obtained by J. Mukai. I researched $[\Sigma^2 \mathrm{P}^n, \Sigma^2 \mathrm{P}^n]$ with J. Mukai[1]. We determined the group structure of $[\Sigma^2 \mathrm{P}^n, \Sigma^2 \mathrm{P}^n]$ for $n \leq 6$ by using the results on $\pi_m(S^n)$, Toda bracket, Whitehead product, Steenrod operation and etc. Furthermore, to research the CW-structure of the homotopy-fibre of the collapsing map, I determined the group structure of $[\Sigma^2 \mathrm{P}^7, \Sigma^2 \mathrm{P}^7]$ [2]. In general, the self-homotopy of a single suspended space is non-commutative. In [5], we proved that $[\Sigma \mathrm{P}^5, \Sigma \mathrm{P}^5]$ is commutative. And we determined the group structure of $[\Sigma \mathrm{P}^5, \Sigma \mathrm{P}^5]$. Moreover, we proved that $[\Sigma \mathrm{P}^6, \Sigma \mathrm{P}^6]$ is commutative, but the group structure of it is unknown.

The Lusternik-Schnirelmann (L-S) category is defined by L. Lusternik and L. Schnirelmann in 1934 as a numerical homotopy invariant of a manifold which gives a lower bound for the number of critical points of a smooth function on a manifold. The definition of the L-S category of a space X is the least number m such that there is a covering of X by $m + 1$ open subsets each of which is contractible in X. This is simple, but it is difficult to calculate L-S category of a space. I research the L-S category of the quasi-projective space Q_n. I gave the condition of n and m that the L-S category of the stunted quasi-projective space $Q_{n,m} = Q_{n,m}$ is equal to 3 with N. Iwase in [3]. As a result of this, we proved that the L-S category of Q_3 is equal to 3. Though Q_3 is a CW-complex of a simple structure with three cells, the L-S category of Q_3 was left unknown. In a proof of it, We researched the relation between the James-Hopf invariant and the cup product on a generalised cohomology theory and used a numerical homotopy invariant called cup-length which gives a lower bound of the L-S category. In a joint research with N. Iwase and K. kikuchi[4], we determined the L-S category of the 10-dimensional rotation group SO(10). We considered using the principal bundle and developed the evaluation of the L-S category of the total space by the product and the L-S category of the structure group. We applied it to the principal bundle $\mathrm{SO}(9) \hookrightarrow \mathrm{SO}(10) \to S^9$. Using the L-S category of SO(9), we proved that the L-S category of SO(10) is equal to 21.