Principal Mathematical Achievements

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From covering to surgery presentations. A *covering presentation* of a 3–manifold M is a knot $K \subset S^3$ and a representation ρ from its knot group onto a finite group of permutations $G \subset \mathrm{Sym}\,(*_1, \ldots, *_n)$. The manifold M is constructed as a covering space of S^3 branched over K with monodromy uniquely specified by the permutation action of G on the sheets given by ρ. Another way of presenting M is via a *surgery presentation*, in which M is presented as an (integral) framed link in S^3. Here M is recovered by (integral Dehn) surgery along the link.

Any compact oriented connected 3–manifold M has both covering presentations and surgery presentations. Roughly speaking, covering presentations have been useful in classical knot theory, while surgery has been useful in quantum topology. I have been interested in translating from covering presentations to surgery presentations. When G is a cyclic group, there is a well-known construction whereby K is untied by surgery, and the surgery link is then lifted to the cyclic cover to yield a surgery presentation for M.

The next case to consider is when G is a dihedral group D_{2n} (n an odd integer). I analyzed the set of covering presentations (K, ρ) with ρ a representation of the knot group onto D_{2n} modulo surgery by unknotted loops whose meridians lie in $\ker \rho$ (these are the loops which will lift to the branched dihedral cover to give a surgery presentation of M). There turn out to be n such equivalence classes, each of which have representatives which lift in a standard way to the dihedral covering, together with the surgery curves in their complement. This was proved for $G = D_6$ and $G = D_{10}$ in my thesis, an upper bound of $2n$ on the number of equivalence classes was established by Litherland and Wallace, and the construction was completed by Kricker and myself in 2007. I have also shown an analogue for the Kirby theorem in this context which tells us when two surgery presentations for the same knot lift to surgery presentations for the same dihedral covering manifold. A corollary to the construction, combining with results of Przytycki, Sokolov, and Sakuma, is that dihedral symmetry of a 3-manifold implies dihedral symmetry of some surgery presentation.

Going further, I have considered $G = A_4$ (symmetries of an oriented tetrahedron). Like the dihedral groups, A_4 is metabelian, and its commutator subgroup (the Klein four group) is small and easy to work with. Combining the method which worked in the dihedral case with some ad hoc steps, I have been able to translate from an A_4 covering presentation of a 3–manifold and a covering link to a surgery presentation.

I am interested in generalizing to all metabelian groups, for which new ideas are necessary.

Quantum topology for branched covering spaces. In my thesis, I used the translation from covering presentation to define a dihedral analogue to Garoufalidis and Kricker's rational Kontsevich integral of a knot when $G = D_{2n}$ for $n = 3, 5$. Its 1–loop part gives a non-commutative analogue to the Alexander polynomial for pairs (K, ρ), and as a 2–loop part I would like to obtain an analogue to the 2–loop polynomial. The construction I have now can be significantly improved using the Kricker and my new translation procedure. The goal is quantum topology for branched covering spaces, and the covering links which they contain.

Yoshida's parametrization of the Prym covering of Hitchin's moduli space. Tomoyoshi Yoshida has set out to complete Atiyah and Hitchin's programme for the abelianization of the $SU(2)$ WZW model, thus determining an explicit basis for the space of $SU(2)$ conformal blocks in terms of Riemann theta functions. The first part of his work involves explicitly parameterizing the Prym cover of the Hitchin moduli space of Higgs bundles. Jointly with S.K. Hansen I used a graph theoretic approach to flesh out and simplify the details in its construction, which is suitable for the more general case where Φ may have poles.

Combinatorics of Jacobi diagrams. A finite type invariant of a link is determined by two pieces of information— a Jacobi diagram (a purely combinatorial object consisting of a formal sum over $\mathbb{Q}$ of uni-trivalent graphs modulo relations), and a representation of a ribbon Hopf algebra (a generalization of a quantum group). A Jacobi diagram consisting of a single connected graph is said to be n–loop if its Euler number is $1 - n$. Spaces of 0–loop Jacobi diagrams correspond to free Lie algebras. I determined a new combinatorial approach to study and enumerate them, with which I could reprove some classical theorems of free Lie algebras.

One important open question in the theory of finite-type invariants is whether they can detect knot orientation. In a joint paper with T. Ohtsuki we proved that the space of 3–loop Jacobi diagrams of odd degree vanishes. This implies that no n–loop finite type invariant can detect knot orientation when $n < 4$.

Self–linking number. Knot theory as a mathematical discipline is often said to have started with Gauss's integral formula for the linking number of two curves. In the later 1950's and early 1960's, this formula was modified to give an invariant of a single closed space curve— *i.e.* a knot (with a framing). There are two main approaches to define the self–linking invariant of a (framed) knot, corresponding to two different ways to compactify the configuration space of pairs of points on the knot. In a project for Dror Bar-Natan in Jerusalem, I proved the known result that these two approaches yield the same invariant.