Results

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A Lagrangian fibration is a map $\pi : (M, \omega) \to B$ from a symplectic manifold such that its general fiber is a Lagrangian submanifold (i.e. $\dim \pi^{-1}(b) = \frac{1}{2} \dim M$ and $\omega|_{\pi^{-1}(b)} = 0$). Completely integrable systems are typical examples. Lagrangian fibrations appear in various areas in mathematics and mathematical physics, such as geometric quantization and mirror symmetry. However the understanding of Lagrangian fibrations is not satisfactory yet, because of the existence of singular fibers.

Lagrangian fibrations of Abelian varieties and theta functions. The theory of theta functions on Abelian varieties is another topic where Lagrangian fibrations play an important role. It is known that a holomorphic sections of an ample line bundle L (and its tensor power L^k) over an Abelian variety A is given by a theta function. Moreover, a natural basis of the space $H^0(L)$ of holomorphic sections of L (or L^k) is related to a certain Lagrangian fibration of A. We studied projective embeddings of A defined by these basis for L^k. For a natural torus action on the ambient projective space, it is proved that its moment map, restricted to A, approximate the Lagrangian fibration of A for large k, with respect to the "Gromov-Hausdorff topology".

Lagrangian fibrations of Kummer varieties and theta functions. A Kummer variety is a quotient of an Abelian variety A by the inverse morphism $(-1) : A \to A$. Holomorphic sections of an ample line bundle on the Kummer variety $A/(-1)$ are induced from theta functions on A. The above result for A is extended to the case of Kummer varieties. Note that the Lagrangian fibration in this case has singular fibers.

Toward a generalization of the approximation theorem. The natural basis of theta functions can be obtained from a Lagrangian fibration using the Bergman kernel, which is an integral kernel of the orthogonal projection from the space of L^2-sections to $H^0(L)$. We apply this construction to general cases and study the asymptotic behavior of the resulting sections under the limit $k \to \infty$. It is proved that these sections have similar properties as for the theta functions.

Hamiltonian minimality and symplectic reductions. We presented a formula for the mean curvatures of Lagrangian submanifolds under a symplectic reduction.

Toric degeneration of flag manifolds and the Gelfand-Cetlin systems. A flag manifold $X = U(n)/T$ has a Lagrangian fibration called the Gelfand-Cetlin system. This is a completely integrable system $X \to \mathbb{R}^{n(n-1)/2}$, and its image is a convex polytope Δ, which is called the Gelfand-Cetlin polytope. Moreover, it is known that X can be degenerated into a toric variety Y corresponding to Δ. We proved that the Gelfand-Cetlin system can be deformed into the moment map of the standard toric action on Y.