Plans of my research.

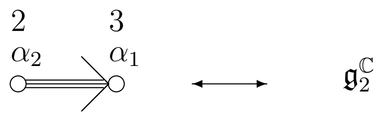
My interest lies in eigenspaces in real simple Lie algebras. Let us give three Examples 1, 2 and 3 of eigenspaces in a real simple Lie algebra $\mathfrak{g}$.

Example 1. Let σ be an involutive automorphism of $\mathfrak{g}$, and let $\mathfrak{h}$ (resp. $\mathfrak{m}$) denote the $+1$-eigenspace (-1-eigenspace) of σ in $\mathfrak{g}$. Then the pair $(\mathfrak{g}, \mathfrak{h})$ is a simple symmetric pair, and $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}$ is its canonical decomposition. Besides, $(\mathfrak{g}, \mathfrak{h})$ gives rise to a simple affine symmetric space G/H, and $\mathfrak{m}$ is assumed to be the tangent space $T_o(G/H)$ at the origin o.

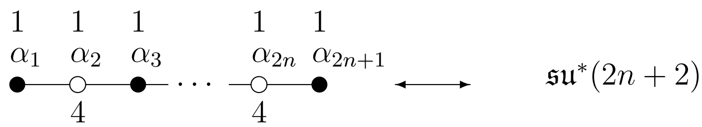
Example 2. Let $\mathrm{ad}_{\mathfrak{g}} X$ denote the adjoint representation of $\mathfrak{g}$, for any non-zero element $X \in \mathfrak{g}$, and let $\ker(\mathrm{ad}_{\mathfrak{g}} X)$ denote the 0-eigenspace of $\mathrm{ad}_{\mathfrak{g}} X$ in $\mathfrak{g}$ (i.e., the kernel of a linear mapping $\mathrm{ad}_{\mathfrak{g}} X$). Then the pair $(\mathfrak{g}, \ker(\mathrm{ad}_{\mathfrak{g}} X))$ gives rise to a symplectic homogeneous space. Conversely, any symplectic homogeneous space of a connected, simple Lie group G is given rise to by such a pair.

Example 3. Suppose that $\mathfrak{g}$ admits a complex structure. Let $\mathfrak{c}$ be a Cartan subalgebra of $\mathfrak{g}$, and let $\mathfrak{g} = \mathfrak{c} \oplus \bigoplus_{\alpha \in \triangle(\mathfrak{g},\mathfrak{c})} \mathfrak{g}_\alpha$ denote the root-space decomposition of $\mathfrak{g}$ with respect to $\mathfrak{c}$. Then for any $C \in \mathfrak{c}$, $\mathfrak{g}_\alpha$ is the $\alpha(C)$-eigenspace of $\mathrm{ad}_{\mathfrak{g}} C$ in $\mathfrak{g}$.

I notice that root-space decompositions of complex simple Lie algebras correspond to Dynkin diagrams and the classification of Dynkin diagrams bring about that of complex simple Lie algebras.



Moreover I notice that (restricted) root-space decompositions of orthogonal, simple symmetric pairs of non-compact type correspond to Satake diagrams and the classification of Satake diagrams bring about that of real simple Lie algebras.



I guess that certain eigenspace decompositions of real simple Lie algebras will correspond to some diagrams and a classification of their diagrams will bring about that of simple symmetric pairs. Our aim in the further is to make diagrams (like Dynkin diagrams, Satake diagrams or Vogan diagrams) which correspond to simple symmetric pairs.