

Plan

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As you know, there are some examples of non-holonomic distributions: ‘ Martinet ’ ‘ contact ’ ‘ Engel ’ and ‘ Cartan ’ are. ‘ Martinet ’ is rank two distribution on three dimensional manifold and has singularities. So we have to take two times Lie-brackets of the sections so that it will be the whole tangent bundle. This is very complicated. On the other hand, ‘ contact ’ is so simple, which is non-degenerate two dimensional distribution of corank one. We only have to take one time Lie-bracket of the sections for it to be the whole space. ‘ Engel ’ is also simple, which is two dimensional distribution of corank two. It will be the whole space by two times Lie-bracket of the sections. ‘ Engel ’ has no local invariant as well as ‘ contact ’. ‘ Cartan ’ is rank two distribution on five dimensional manifold. It will be the whole tangent bundle by two times Lie-brackets of the sections. If we take one time Lie-bracket, it becomes three dimensional. And it becomes five dimensional space (the whole space) by one more bracket. This is very interesting structure. Its automorphism group makes a Lie group of dimension not greater than 14, and if the maximal dimension attained, then the automorphism group is locally isomorphic to the exceptional Lie group G_2 .

My plan is as follows:

1. Classification of the infinitesimal automorphisms of an homogeneous Engel sub-Riemannian structure.
2. Classification of the infinitesimal automorphisms of an homogeneous contact sub-Riemannian structure which is of dimension not greater than $(n + 1)^2$.
3. Classification of the infinitesimal automorphisms of an homogeneous Cartan sub-Riemannian structure.
4. Construction good examples of geodesics on sub-Riemannian manifold equipped with subbundle of corank greater than 2.