Results

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My subject is a sub-Riemannian geometry. A sub-Riemannian geometry is a manifold endowed with a distribution and a fibre inner product on the distribution. A distribution here means a linear subbundle of the manifold. This Riemannian fibre metric is called a Carnot Caratheodry metric, which is related an Optimal Control Theory. Now we suppose that a distribution is bracket generating or non-holonomy (i.e., the collection of all vector fields generated by Lie brackets spans the whole tangent bunndle). We study a local classification and the geodesics of sub-Riemannian manifolds with non-holonomoic distributions.

- Classifications

The structures of tangent distributions on manifolds are related with sub-Riemannian manifolds. There are some typical non-holonomic distributions, for examle, Martinet distribution, contact distribution, Engel distribution and Cartan distribution. Now we define the fibre inner product on such a non-holonomic distribution, that is a sub-Riemannian manifold. Then there naturally occured the local classification problems. First we choose a contact distribution, which is simple but non-trivial and interesting class. In [3] we study the structures of infinitesimal automorphisms of a homogeneous sub-Riemannian contact manifold from the view point of nilpotent geometry. We have that the infinitesimal automorphisms is of finite dimension less than or equal to $(n+1)^2$ if the dimension of manifold is $(2n+1)$. We then completely determine the structures of the infinitesimal automorphisms, which attain the maximal dimension . We also discribe the standard concrete sub-Riemannian manifolds on which these Lie algebra shaves are realized. Now we study infinitesimal automorphisms of homogeneous Engel sub-Riemannan manifolds. We see a class of infinitesimal automorphisms of Engel sub-Riemannian structures which is isomorphic to infinitesimal automorphisms of contact sub-Riemannian structures using its jet spaces. However there may be another class of infinitesimal automorphisms of Engel sub-Riemannian structures, so we are studying more. Moreover we study the invariants of "sub-Riemannian Cartan structures" (as you know "Cartan distribution" has some invariants).

- Geodesics

Riemannian geometry tells us that a minimizer (i.e., a shortest path) between two points of a Riemannian manifold is a geodesic, and the geodesics are characterized to be the curves satisfying the geodesic equation expressed in local coordinates. Conversely, every geodesic is locally length minimizing. In the formulation of symplectic geometry, the geodesics are the projections to the base manifold of the integral curves of the Hamiltonian vector field defined on the cotangent bundle. Now in sub-Riemannian geometry, it is also of fundamental importance to study minimizers between two points of a sub-Riemannian manifold. However, contrary to the Riemannian case, this problem is very subtle, mainly because the space of all integral curves of the distribution joining two points may have singularities, which makes difficult to apply directly the method of variation to the sub-Riemannian case. For a sub-Riemannian manifold we define a *normal geodesic* to be an projection of integral curve of the Hamiltonian vector field associated to the Hamiltonian function associated with the subriemannian metric. Then, as in Riemannian geometry, a normal geodesic is locally a minimizer. However, R. Montgomery and I. Kupka discovered that there exists a minimizer which is not a normal, and hence called it abnormal. The appearance of abnormal minimizers is a surprising phenomenon never arising in Riemannian geometry but peculiar to sub-Riemannian geometry. A rigorous application of the Pontryagin Maximum Principle of Optimal Control Theory to sub-Riemannian geometry shows that a minimizer of sub-Riemannian manifold is either a normal or an abnormal. We then consider this problem in a concrete case of the standard Cartan distribution. We carry out detailed computation of minimizers, which will well illustrate how normal and abnormal geodesics appear in sub-Riemannian geometry.