Research statement (project)

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GKM manifolds are main research objects during my researcher period. In particular, I focus on studying the Equivariantly formal spaces in GKM manidolds. Here, the *GKM manifold* is an even dimensional manifold with a torus action such that the set of its fixed points and 1-dimensional orbits becomes a graph, i.e., vertices and edges (it is possible to be legs), in its orbit space. This definition is so simple; therefore, this class consists of the vast class in all manifolds. For example, torus manifolds, which are the part of GKM manifolds, can be constructed by the gluing of two manifolds such that its orbit space is the connected sum of a simple n-dimensional convex polytope and any n-dimensional manifold. From this point of view, all manifolds can appear in the class of torus manifolds. So, torus manifolds (GKM manifolds) are very complicated class. As a nice class in GKM manifolds, Goresky-Kottwitz-MacPherson (GKM) suggested the Equivariantly formal space in 1998. Here, the *Equivariantly formal space* is a GKM manifold M with T-action whose equivariant cohomology satisfies $H_T^*(M) \simeq H^*(M) \otimes H^*(BT)$. This assumption seems to be an artificial assumption; however, GKM showed that this class is the necessary and sufficient class which satisfies that an equivariant cohomology $H_T^*(M)$ corresponds with an ordinary cohomology $H^*(M)$ by the Koszul duality functor. Therefore, the Equivariantly formal space is an important class in GKM manifolds.

In fact, Masuda-Panov (resp. Konno, Harada, Holm and Proudfoot) showed very beautiful results on equivariant cohomologies and geometric properties of Equivariantly formal, torus manifolds (resp. hypertoric varieties). Moreover, the natural T-action on Grassmannian is the Equivariantly formal space, and there are many beautiful researches on it, for example the equivariant Schubert calculus. Due to such beautiful results, we can expect to get the beautiful results on the Equivariantly formal spaces. However, it seems to be that there are few studies except the above classes. Hence, one of the interesting research of this area is to find the new class in the Equivariantly formal spaces. During my researcher period, I try to find the class which corresponds with the generalization of hypertoric varieties (see the previous project for detail).

Next, I would like to study extended actions of GKM manifolds more deeply. I studied on the extended actions of torus manifolds in the papers (4), (6), (7), (9). In my feeling, we can get more deep results by the different direction from these papers. Moreover, because the extended action is the subgroup in $\mathrm{Diff}(M)$, I would like to compute a ring structure of $H^*(\mathrm{BDiff}(M);\ \mathbb{Q})$.

Third, I would like to find the application to combinatorial theory by using combinatorial objects defined from GKM manifolds (graph, polytope, matroid, etc). The biggest achievement of such application is the g-theorem by Stanley; however, there should be more applications. For example, we can regard the above extension problem (in particular, on quasitoric manifolds) as the problem of projections form simple, convex polytopes to lower dimensional polytopes. Moreover, we might be able to apply to a coloring problem on graphs by using a coloring on GKM graphs which defined by geometric structures like omniorientations.

Finally, if the GKM manifold is not Equivariantly formal, then its equivariant cohomology does not corresponds with the ordinary cohomology. Therefore, to study all GKM manifolds, it might be natural to consider the corresponding model instead of the cohomology. If I have a chance, I would like to study such model.