Research statement (results)

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I have researched transformation groups on manifolds from topological point of view. My research can be divided into the following three researches. The numbers which appear in the followings correspond with the numbers of "1.ACCEPTED PAPERS" and "2.PREPRINTS" in the "List of publications and preprints".

0.1. Toric topology. Toric topology is the study of algebraic, combinatorial, differential, geometric, and homotopy theoretic aspects of a particular class of torus actions, whose quotients are highly structured.

The papers (5) and (8) correspond with the study of combinatorial aspect. In (5), I defined a hyper torus graph (HTG) as a new class of graph which is similar to the GKM graph and studied their equivariant cohomology ring structure. We regard the result on equivariant cohomology rings of HTG as a generalization of that of hypertoric varieties. In (8), I showed surgery operations ($\flat$, $\sharp^e$, $\sharp^{eve}$) in 3-dimensional small covers can be constructed from the other surgery operations ($\sharp$, $\natural$). These operations can be regarded as the combinatorial operations on 3-dimensional polytopes. From this result, we can improve the Nishimura's and Lü-Yu's construction theorem of 3-dimensional small covers, i.e., 3-dimensional small covers can be constructed by using basic small covers and surgery operations.

The papers (4), (6), (7), (9) and (10) correspond with the study of geometric (transformation theoretic) aspect. In (4), (6), (7), I classified homogeneous torus manifolds and torus manifolds with codimension one extended actions. In particular, such (quasi)toric manifolds (some nice class in torus manifolds) are complex projective bundles over product of complex projective spaces, and all such extended actions are induced from such actions on moment-angle manifolds. Moreover, in (9), Choi and I determined topological types of such torus manifolds (only on the restricted cases). They are completely determined by their cohomology rings and characteristic classes. This result gives counter examples of cohomological rigidity of torus manifolds whose orbit spaces are homotopy cells. In (10), Lü and I showed the construction theorem on projective bundles over 2-dimensional small covers.

0.2. Classification of the compact Lie group action. In (3), I completely classified compact Lie groups, which act on a rational cohomology complex quadric M (a manifold whose rational cohomology is isomorphic to $Q_{2n} \cong SO(2n+2)/SO(2n) \times SO(2)$) with codimension one principal orbits, and I also classified topological types of M. As the results, there are eight different pairs of actions and manifolds. In particular, there is exactly one manifold whose topological type is not the complex quadric, and one group action on the complex quadric which does not appear in the results of Kollross (he classified K, which is a subgroup of G and acts on G/H with codimension one orbits).

0.3. Smooth non-compact Lie group action. In (1), I constructed infinitely many smooth $SL(m, \mathbb{H}) \times SL(n, \mathbb{H})$-actions on $S^{4(m+n)-1}$ by using smooth $\mathbb{R}^2$-actions on S^7.

In (2), I proved (well-known facts) $\mathbb{C}P(2)/\mathrm{conj} \cong S^4$ directly by using the application of the Uchida's method, and by using this fact I could construct a continuous $SL(3, \mathbb{R})$-action on S^4 whose restricted $SO(3)$-action is the conjugated action.