

Based on the brane world scenario, there are many studies on higher-dimensional objects. The study of *squashed Kaluza-Klein black holes* [1-3,7,9,10] is one of examples. I expect that these black holes have the rich properties never shown until now. To discuss these variety of properties, I will consider **inner structures of squashed Kaluza-Klein black holes** as the first step. My preliminary research shows that there are two types of static squashed Kaluza-Klein black holes related to the shapes of the curvature singularity. I expect that we have some method to clarify the difference of these two types of black holes, if the solution describes our real universe. Next, deeply related to previous research, I will discuss **verification of extra dimension by squashed Kaluza-Klein black holes**. I will apply squashed Kaluza-Klein black holes to the outer region of the gravitational source and consider the evaluating of the size of the extra dimension by the effect around the the Earth and the Sun. Finally, I will consider **stabilities of squashed Kerr-Gödel black holes** against several perturbations. I expect that this research may help us to clarify which black hole solutions are suitable for the discussions of the higher-dimensional spacetime.

**Inner Structures of Squashed Kaluza-Klein Black Holes** I have studied charged static squashed Kaluza-Klein black hole solutions which asymptote to the locally flat spacetime, i.e., a twisted constant  $S^1$  bundle over the four-dimensional Minkowski spacetime, at the infinity. These solutions are specified by the three parameters, namely, the mass, the electric charge and the size of the extra dimension. Recently, I have obtained two types of Kaluza-Klein black hole solutions related to the shapes of the curvature singularities. When the electric charge is smaller than the critical charge specified by the size of the  $S^1$  fiber at the spatial infinity, there is a point singularity where the spatial cross section of the spacetime shrinks to a point. On the other hand, when the electric charge is larger than the critical charge, there exist a stretched singularity where the the spatial cross section of the spacetime is stretched infinitely along the  $S^1$  fiber.

I expect that the difference of these two types of singularities has an effect on the global spacetime geometry. I will consider the motions of a test particle in these Kaluza-Klein black hole spacetimes and discuss the relation between the behaviors of a test particle and two types of curvature singularities. I will also extend these discussions to other squashed Kaluza-Klein black holes.

**Verification of Extra Dimension by Squashed Kaluza-Klein Black Holes** In the brane world scenarios, one of the interesting problems is a verification of the extra dimensions from the various physical phenomena in higher-dimensional spacetimes. In such phenomena, I focus on the Gravity Probe B experiment and the perihelion precession of the Mercury as the first step. Here, the Gravity Probe B satellite moves along a stable circular orbit around the Earth and collects data about two gravitational effects, the geodetic effect and the frame dragging effect. I focus on the geodetic effect which is the leading effect and estimated by the four-dimensional vacuum static Schwarzschild black hole. I expect that the geodetic effect is suffered from a correction by the existence of extra dimensions.

Thus I will apply the five-dimensional vacuum static squashed Kaluza-Klein black hole solution to the outer region of the Earth. I will estimate the size of the compact extra dimension by the higher-dimensional correction to the geodetic precession angle with the Gravity Probe B experiment. Even if, the result of the Gravity Probe B measurement gives the predicted value with an expected accuracy, the accuracy of the observation gives an upper limit of the size of an extra dimension. I will compare this correction with that to the perihelion precession of the Mercury by applying the five-dimensional vacuum static squashed Kaluza-Klein black hole solution to the outer region of the Sun.

**Stability of Squashed Kerr-Gödel Black Holes** Recently, I have constructed squashed Kerr-Gödel black hole solutions in the five-dimensional Einstein-Maxwell system with a Chern-Simons term [2]. These solutions describe rotating black holes in the supersymmetric rotating Gross-Perry-Sorkin (GPS) monopole spacetime. This background geometry is everywhere nonsingular and there is an ergoregion with a shape of shell.

To discuss the stability of the rotating GPS monopole spacetime, I will consider massless scalar fields in this background as the first step. I will discuss the stabilities against such field perturbations, due to the combined effect of the superradiant mechanism and of the presence of an effective reflective wall sourced by the Kaluza-Klein momentum of the mode along the extra dimensional direction. I will also consider the quasinormal frequencies for the scalar field perturbations of the rotating GPS monopole background. I will extend these discussions to squashed Kerr-Gödel black holes which admit two disconnected ergoregions, an inner and an outer ergoregions.