Research Statement

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Invariants of 2-bridge knots (Publications and Preprints [1, 2, 3, 4]): For each pair of coprime integers (p, q), there corresponds a 2-bridge knot, which has two kind of knot diagrams; One is a knot diagram called Conway's normal form, which is given by using a continued fraction expansion of p/q, and another is a knot diagram called Schubert's normal form, which is given directly from p and q without using a continued fraction expansion. In , G. Burde gave a formula for the Conway polynomial of 2-bridge knots by means of a continued fraction expansion of p/q. On the other hand, in , R. Tuler gave a formula for the first coefficient of the Conway polynomial by means of a continued fraction expansion. He also gave a formula by means of a continued fraction expansion, which is different from Burde's. Furthermore, by using these formulas, Tuler gave an arithmetic relation about Rademacher-Dedekind homomorphism, which is an important subject in number theory. To extend his work I studied the second coefficient of the Conway polynomial, which is an example of the Vassiliev invariant of order two called the Casson knot invariant. In [1], by using the Gauss diagram of Schubert's normal form, I gave a formula without using a continued fraction expansion. In [2], I extended this formula by introducing the map which is useful to handle the Gauss diagram of Schubert's normal form. In [3], I also gave a formula for the Vassiliev invariant of order three without using a continued fraction expansion. In [4], I studied finite type invariants of PSL(2,Z), which is introduced by the fact that Conway's normal form corresponds to an element of SL(2,Z). So far I have determined the invariant whose order is less than four and I give an arithmetic relation between that invariant and the invariant of 2-bridge knots.

Invariants of ribbon knots and their 2-fold branched covers (Publications and Preprints [5, 6, 7, 8]): In 1966, R.H. Fox and J.Milnor defined the equivalence relation on knots called cobordance. The set of all knots is required as a group by cobordance. The knots which are the identity element is called the slice knots. It is a basic problem to determine the slice. But this problem contains many difficulties relating to 4-dimensional topology. So Fox and Milnor defined the ribbon knots to approach the slice knots. By the definition of ribbon knots, they are slice knots. One of aims in investigating ribbon knots is to approach this famous problem: “Are slice knots ribbon knots?” Fox and Milnor gave the following famous property; the Alexander polynomial of slice knots is of the form f(t)f(1/t), where f(t) is a Laurent polynomial. To have other necessity conditions for ribbon knots I have been investigating invariants of ribbon knots. In [5], I studied the Jones polynomial of ribbon knots and I gave an explicit formula for coefficients of the Jones polynomial of ribbon knots and derived some necessity conditions for ribbon knots from the formula. In [6], to investigate the complexity of a ribbon knot, I defined the notion of the ribbon number as the minimal number of ribbon singularities needed for a ribbon disk bounded by the ribbon knot. This obvious measure of a knot's complexity is often hard to determine. In fact, even in a simple case of ribbon knots, its ribbon number is hard to determine. Then I estimate the ribbon number by using a formula in [5]. In particular, I determined that the ribbon number of the Kinoshita-Terasaka knot is two. In [8], Y. Tsutsumi and I investigated the ribbon number by using the genus and the crosscap number.

In 1978 Montesinos showed the 2-fold branched cover along a ribbon knot of 1-fusion is a homology 3-sphere bounding a contractible 4-manifold of Mazur type. In [7], I give an explicit formula for the Casson invariant of a homology 3-sphere of Mazur type.

A program to find ribbon knots (Publications and Preprints [9, 10]): I made a program to find ribbon knots in the knot table and had candidates of counter examples of the slice-ribbon problem mentioned above. Moreover, I derived some conditions related to the unlinking number of links to have a more precise program.