Research Plans

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The objects of my research are holomorphic families of Riemann surfaces. Especially, I study the following two problems.

1. On a Teichmüller disk

Purpose. For a holomorphic family (M, π, B) of closed Riemann surfaces of genus g over a hyperbolic Riemann surface B, we obtain a holomorphic mapping $\widetilde{J}$ from the universal covering $\widetilde{B}$(=the unite disk) of B to the Teichmüller space of genus g. For the image $\widetilde{D} = \widetilde{J}(\widetilde{B})$, we have the following problem: Does $\widetilde{D}$ become a Teichmüller disk? In this research, we prove the following claim:

Claim. If the universal covering transformation group Γ of B is of divergence type, then $\widetilde{D}$ does not become a Teichmüller disk.

Contents. We will show two claims (i) and (ii) as follows:

(i) $\widetilde{D}$ becomes a disk.

(ii) $\widetilde{D}$ does not become a Teichmüller disk.

To prove (i), suppose that $\widetilde{D}$ does not become a disk. Then $\widetilde{D}$ has a boundary component which is contained in T_g. Since Γ is of divergence type, there exists a boundary point in $\partial\Delta$ such that the image of the point under $\widetilde{J}$ is in the boundary component. Next, by investigating the properties of holomorphic mappings, we have a contradiction.

To prove (ii), suppose that $\widetilde{D}$ is a Teichmüller disk, and consider the projection $\widetilde{D} \to M_g$ to the moduli space of genus g. At this time, I hope that I use an idea of Veech.

2. On an estimation of the number of holomorphic sections

Purpose. It is important to determine the number of holomorphic sections of holomorphic families. In this research, I investigate examples of holomorphic families and try to estimate the number of their holomorphic sections.

Contents. First, I try to obtain the best estimation Riera's example. At the same time, I investigate many examples and conjecture what the number of their sections depends on. Finally, I prove the conjecture.