Plans of my study

Takanari Saotome

As a plan of my study, I will study a geometry of s.p.c. manifolds from two aspects. First one is a continuation of the study of J-holomorphic mappings for s.p.c. manifolds which has been proceeded until now by me and my supervisor, Professor M.Itoh. It is a hopeful issue to construct a theory analogous to that of J-holomorphic curves for symplectic manifolds. Another one is a study of relevant even-dimensional geometries. Since s.p.c. manifolds deeply relate with even-dimensional geometry through the Boothby-Wang fibration structure and a boundary structure, investigations of these structures and relationship with a s.p.c. manifold will not only be a logical nonsense but also be linked to developing a new method to study symplectic geometry.

In the study of J-holomorphic mappings, the first step to make the theory available is to examine the moduli space of all J-holomorphic mappings. Particularly, it is a crucial problem to make clear whether the moduli space is a compact finite dimensional manifold or not and to clear this I will take means which use Fredholm theory of the defining equation of J-holomorphic mappings and the bootsstrapping theory for s.p.c. manifolds. It will also be important to investigate analytic properties of individual J-holomorphic mapping and geometrical properties of its self-intersections in order to compactify the moduli space and to construct certain invariants. I intend to study these topics with comparing it by the symplectic geometry. Present problems in my study are mainly the followings.

(1) Does the equation which defines J-holomorphic mappings have the Fredholm property? (The sub-elliptic propertiy of it has been proved and so relationship of the equation with the implicit function theorem is a focus.)
(2) The bootsstrapping theory: Since this theory relates with the dimension of unlying manifolds, it is not trivial whether it holds or not like as in the symplectic case. I expect smoothness of the derivative of a mapping by the characteristic direction is a key to solve this problem.
(3) bubbling phenomena: In the symplectic case, because of conformal invariance of the energy functional a limit of J-holomorphic curves may happen bubbling off. On the other hand, it is natural to consider that J-holomorphic mappings for s.p.c. manifolds are mappings between s.p.c. manifolds and their energy functional do not have conformal invariance. An observation of what happens in this case is needed.

As the second plan of study, I am interested in a relation between a s.p.c. manifold and even-dimensional manifolds relevant to it. The fibration structure and a boundary structure are respectively related to an orbifold and a cone manifold, so it is expectable that this study is linked to study of a geometry of even-dimensional manifolds with singularity.

(4) In the case that an s.p.c. manifold admits a Boothby-Wang fibration structure, I would like to compare the geometry of s.p.c. manifold and that of base manifold.
(5) I would like to consider the singularity from an aspect of geometry of the s.p.c. manifold, when a s.p.c. manifold is a link of isolated singularity in a complex manifold.
(6) I would like to compare a geometry of non-compact Kähler manifold with cylinderical ends and a geometry of a s.p.c. manifold appeared as the section of a end.
(7) Considering manifolds with isolated singularity as a panctured manifold, I would like to study a geometry of s.p.c. manifolds with singularity. (It may be studied using the method like as in [2].)