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# A DESCRIPTION OF RESEARCH

I carried out researches of low-dimensional manifolds by using knot theory so far. Knot theory is the theory for the mathematical study of knots and links to investigate knotted simple closed curves in 3-space. I explain my previous research for the last 12 years.

## Previous research

### Period 1. 1996-1999

I studied knot cobordism in terms of the properties of quasipositive knots. Particularly, I showed the existence of an infinite family of linearly independent knots with arbitrary gaps between 4-genera and topological 4-genera by showing special properties of quasipositive knots concerning knot cobordism. the maximal Thurston-Bennequin number is an invariant of knots and links derived from 3-dimensional contact geometry. I gave certain formula concerning Kauffman polynomials and Maximal Thurston-Bennequin numbers of positive knots and links. Moreover, I generalized the result to alternating knots and links.

### Period 2. 2000-2001

By studing the properties of the Jones polynomials of symmetric unions of knots, I gave an infinite family of counterexamples for a conjecture of T. Fiedler, given in 1999, concerning the cobordism invariance of the Jones polynomial of a knot. Furthermore, by developing the research, I gave a classification of 2-knots (embedded spheres in 4-space) associated with symmetric unions.

### Period 3. 2002

I researched on a linear representation of the fundamental group of the knot complement of a knot. The $A$-polynomial of a knot is an invariant derived from all $SL(2, C)$-representations of the fundamental group of the knot complement. D. Cooper and D. Long conjectured that a knot with trivial A-polynomial is the unknot. This problem was known to be true for knots except satellite knots with trivial Alexander polynomials at that time. I showed the existence of an infinite family of prime satellite knots with the trivial Alexander polynomials and the nontrivial $A$-polynomials.

### Period 4. 2003-2004

I studied on the colored Jones polynomials of knots and links. Now it is considered to be important to find a formula for the colored Jones polynomial because it contributes to investigating the Kashaev-Murakami-Murakami volume conjecture (that states how to evaluate the Gromov invariant of a knot from its colored Jones polynomial). We obtained a formula for the $N$-colored Jones polynomial of a double of a knot $K$ in terms of the colored Jones polynomial of $K$. It generalizes Masbaum's formula for $K$ being the unknot. As a corollary, we showed that if the Kashaev-Murakami-Murakami volume conjecture for untwisted positive (or negative) doubles of knots is true, then the colored Jones polynomial detects the unknot.

### Period 5. 2005

I investigated a relationship between knot cobordism and a clasper. A clasper is an important object in a study of finite type invariants. I showed that if a knot $K$ is obtained from $K'$ by using a graph clasper whose all components have the positive first Betti numbers, then there is a ribbon concordance from $K'$ to $K$.

### Period 6. 2006

I confirmed a question of E. Ferrand which suggests a relationship between HOMFLY polynomials and Kauffman polynomials. By collaborating with A. Stoimenow and D. Matei, we gave counterexamples for a problem of Przytycki (Kirby's problem 1.91(2)) concerning the colored Jones polynomials and mutations of a knot. I have obtained a gauge-theory free proof for the existence of an exotic structure of a certain Casson handle using Rasmussen's invariant derived from Khovanov theory.

**Period 7. 2007**

We have studied on smooth structures on 4-manifolds. By using Rasmussen's invariant, we have shown the existence of an exotic smooth structure on any noncompact, connected smooth 4-submanifold of 4-space. Moreover we have obtained the same result for any noncompact, connected smooth 4-submanifold of $\sharp nCP^2$ for any positive integer $n$ by using Donaldson theory. We obtained a formula of the maximal Thurston-Bennequin number for a double of a knot. We studied the distance between a knot and the set of positive knots by using properties of Rasmussen's invariant.

**Period 8. 2008-**

First, we have investigated spatial graphs by considering special embedding which consists of round arcs in regular circles. In the case of knots, we define an invariant by taking the minimal number of round arcs. We found several relations between the invariant and the other numerical invariants of knots. In particular, we show that the number is three if and only of the knot is a trefoil knot.

We have investigated the Gordian distance for knots by using the signature, the Jones polynomial, the Q polynomial and Rasmussen's invariant.

By using Rasmussen's $s$-invariant derived from Khovanov theory, we give a gauge-theory free proof for the existence of an infinite family of Casson handles. In particular, I have obtained a formula of slice genera for a certain infinite family of Casson handles.