Research plan

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Let G be a connected reductive algebraic group over an algebraically closed field $\Bbbk$ and $\mathfrak{g}$ the Lie algebra of G. Let R be the root system of G. It is known the following;

(1) The number of nilpotent $\mathrm{Ad}(G)$-orbits in $\mathfrak{g}$ is finite.

(2) If X is a nilpotent element in $\mathfrak{g}$, then $2 \dim \mathcal{B}_X + \dim \mathrm{Ad}(G)(X) = \# R$, where $\mathcal{B}_X$ is the variety of Borel subalgebras containing X.

If $\Bbbk$ has bad characteristic for G, these were proved by case-by-case considerations. So we are interested in giving a conceptual proof of them for arbitrary characteristic.

First, we explain (1) in detail. The fact (1) implies several results: the existence of regular nilpotent orbits, the existence of Richardson orbit corresponding to any parabolic subgroup of G, and so on. Therefore it is important to find a direct proof of (1) even if we cannot know the exact number of nilpotent orbits in $\mathfrak{g}$ through the proof. If $\Bbbk$ has good characteristic for G, the fact (1) was proved by Richardson about forty years ago; furthermore we know the number of nilpotent orbits in $\mathfrak{g}$ through the Bala-Carter theorem and Pommerening's theorem. However If $\Bbbk$ has bad characteristic, the number of nilpotent orbits in $\mathfrak{g}$ was calculated by case-by-case considerations and concluded (1); the calculation needed a computer if G is almost simple of type E_7 or E_8. So we want to give a noncomputational proof of (1).

On the other hand, the fact (2) implies that the dimension of each $\mathrm{Ad}(G)$-orbit in $\mathfrak{g}$ is even, and we can see the construction of the orbital varieties in $\mathfrak{g}$, Moreover we can also get the following important result by using the theory of Springer representations: the nilpotent orbits can be parametrized by irreducible representations of the Weyl group. If $\Bbbk$ has good characteristic for G, the fact (2) can be proved by Pommerening's theorem. However, if $\Bbbk$ has bad characteristic, we need case-by-base considerations. By the way, it is known that (2) implies (1). So we desire a direct proof of (2).

In addition, we want to make efforts to the research of new problems in the representation theory of algebraic groups. The theory of various fields might entered, so we should study to understand the cutting-edge research. However there is a lot of rooms for advancement, so we want to try them.