Research Plan

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My research interests lie in the area of 3-dimensional topology, hyperbolic geometry, character varieties and A-polynomials. Especially I study the deformation space of $\mathrm{PSL}(2,\mathbb{C})$ (or $\mathrm{SL}(2,\mathbb{C})$) representations and its application to 3-manifolds. I am also interested in invariants associated with $\mathrm{PSL}(2,\mathbb{C})$ representations e.g. volume and Chern-Simons invariants.

Ideal points In [2], I showed a method for finding ideal points of the character variety of a 3-manifold with torus boundary. But there are many ideal points which can not be detected by my method. My aim is to improve it to find more ideal points. For example, the ideal point on which some ideal tetrahedra do not degenerate could not be detected by my method. For instance, if the volume of the representation is not zero at an ideal point, there exists an ideal tetrahedron which does not degenerate. In this case, it might correspond to an ideal point which induces a torus splitting. Indeed $(-2,p,q)$-pretzel knot complements have ideal points corresponding to torus splitting for some p and q and the volume of representations converges to the volume of the hyperbolic part of the splitting. One of my aim is to extend the method for these cases. If we can find more ideal points, we obtain a good estimate of the Culler-Shalen norm. By using the estimate, I want to obtain results on cyclic or finite surgery as in the paper [3].

I also plan to investigate a method for manifolds with more than one toral boundary components. If we do Dehn filling on toral boundaries except one component, we obtain a family of knot complements. I want to find a way to calculate ideal points for this family of knot complements. For example, it might be interesting to apply it to the link complement used in [3] which produces $(-2,p,q)$-pretzel knots.

More challenging problem is to investigate theory of compactification of $\mathrm{PSL}(2,\mathbb{C})$-representation space of surfaces or 3-manifolds with higher genus boundary. In [1], I give a parametrization of the $\mathrm{PSL}(2,\mathbb{C})$-representations in terms of holonomy and twist parameters associated with a pants decomposition. By definition, it is natural to ask a relationship with the Dehn-Nielsen coordinate system of measured laminations. I remark that Morgan and Shalen gave a construction of compactification of $\mathrm{PSL}(2,\mathbb{C})$-representation space. Let M be a 3-manifold with boundary whose genus is greater than one. Pick a pants decomposition of ∂M, we can construct an ideal triangulation associated with the pants decomposition. At each ideal point of the $\mathrm{PSL}(2,\mathbb{C})$-representation space, we obtain a family of squares on each ideal tetrahedron as in the paper of Yoshida. As restriction of these squares on the boundary, we obtain a family of simple closed curves on the boundary ∂M. It might be natural to ask the relationship with compactification by projective measured laminations.

Exceptional surgery As a sequel of the paper [3], I plan to study Seifert surgeries of $(-2,p,q)$-pretzel knots. In the paper, we showed that there are few candidate for exceptional surgeries by using 6-theorem. On the other hand, Ben Abdelghani and Boyer studied Culler-Shalen norm for Seifert surgeries. I want to apply their results to these candidates of Seifert surgeries and obtain results on Seifert surgeries of $(-2,p,q)$-pretzel knots.