Research statement (projects)

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During the OCAMI researcher in 2010, I will continue the research that related to Toric Topology. In view of the recent progress of my work, the following two subjects will be focused on.

(1) In Differential and Algebraic Geometry, there is the object called a toric hyperKähler manifold. This is the hyperKähler analogue of toric manifolds defined by taking the hyperKähler quotient of the torus action on the quaternionic space. This space can be described by the combinatorial data of hyperplane arrangement like the toric manifold. By its definition, this spce is the $4n$-dimensional manifold with n-dimensional torus action, and very different from toric manifolds as a space; however, very similar phenomena occur between them. For instance, their equivariant cohomology rings can be described by the face ring of their combinatorial objects, or the equivariant cohomological rigidity holds. So, the problem "How do we unify the resemble phenomena seen in these objects?" is one of the natural problems. In 2002, Hausel-Sturmfels gave one answer to this problem by introducing the Lawrence toric manifold. However, because (smooth) toric hyperKähler manifolds are not a big class (up to diffeomorphism), there is a possibility to be able to extend their results to wider class. Moreover, recently, quasitoric manifolds which correspond with topological counterpart of toric manifolds have been widely studying. In my recant research (I am writing the paper now), I define the concept which corresponds with their topological counterpart. These concepts would be one of the answers (but not final) which I asked topological generalization of toric hyperKähler manifolds in the previous plan. Moreover, these classes include not only toric hyperKähler and toric manifolds but also quasitoric manifolds. So, the first goal is to construct the theory which unify the toric hyperKähler manifolds and toric manifolds from the topological point of view, and get the topological generalization of the results of Hausel-Sturmfels. Moreover, I will find applications to toric manifolds (in particular, cohomological rigidity problem), and study the relation to Preprint (6) (which has been tried to unify more general class from the GKM graph point of view).

(2) The second is to study about making the geometric properties from the equivariant cohomology. According to the recent work of M.Masuda and Preprint (11), we see that the equivariant cohomology can reconstruct some geometric properties of spaces. So, I would like to study about "What kind of geometric structures can be reconstructed by the equivariant cohomology?". This kind of problem is classical and central in geometric topology. In the development of recent Toric Topology, I expect that it will obtain some beautiful answer. Moreover, the research on the extension of the group action written in *the research plan in 2009* can be caught in this frame. Masuda defines the root system in the equivariant cohomology of toric manifolds in his recent research, and shows that informations about their extended actions can be caught by using it. According to his research, it can be said that informations about the extended actions also appear in the equivariant cohomology. To study extended actions is to find the essential action. I think the basic geometry of the space comes from the transformation group on the space, as in the F.Klein's Erlangen program. Therefore, to obtain informations of extended actions from the equivariant cohomology can be regarded as the part of this second research; so, for example, I would like to consider whether we can define the root system for wider classes (e.g. equivariantly formal GKM spaces).