Research statement (achievements)

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I have researched transformation groups on manifolds from topological point of view. My research can be divided into the following three researches. The numbers which appear in the followings correspond with the numbers of "1.ACCEPTED PAPERS" and "2.PREPRINTS" in the "List of publications and preprints" (also see *the research result in 2009*).

1.Toric topology. Toric topology is the study of algebraic, combinatorial, differential, geometric, and homotopy theoretic aspects of a particular class of torus actions, whose quotients are highly structured.

First, I summarize my achievements which I have gotten until the year before last in chronological order. In (6), I defined the new class of graphs called hypertorus graph like GKM graphs, and determined its graph equivariant cohomology ring structure. In (4), (7), (8), I classified homogeneous torus manifolds and torus manifolds with codimension one extended actions. In (9), I classified topological types of the manifolds which appeared in (4), (7) and (8) (but of the restricted classes), with S. Choi in OCAMI. As one of the conclusion of this classification, we get the negative answer of the cohomological rigidity problem (mentioned below) of torus manifolds whose orbit spaces are homotopy cell. In (5), the operations ($\flat$, $\sharp^e$, $\sharp^{eve}$) in 3-dimensional small covers can be constructed from the other operations ($\sharp$, $\natural$). In (10), I studied the projective bundles over small covers with Z. Lü in Fudan University.

Last year, in (11) and (12), I proved the equivariant cohomological and cohomological rigidity of toric hyperKähler manifolds. The cohomological rigidity is the property that the cohomology ring determines the geometric structure. The equivariant cohomological rigidity of toric manifolds was proved by M. Masuda, but the problem of non-equivariant cases is still open. We can also naturally ask whether the cohomological rigidity holds for the other classes. I showed that in (11) the equivariant cohomology determines the hyperhamiltonian structure, and in (12) the cohomology (and dimension) determines the diffeomorphism types of toric hyperKähler manifolds (hyperKähler analogue of toric manifolds).

2.Classification of the compact Lie group action. In (3), I completely classified compact Lie groups, which act on a rational cohomology complex quadric M (a manifold whose rational cohomology is isomorphic to $Q_{2n} \cong SO(2n+2)/SO(2n) \times SO(2)$) with codimension one principal orbits.

The above (4), (7), (8), (11) are also the research about the classification of group actions.

Moreover, in Proceedings (8) I studied the classification of 8-dimensional manifolds with $SU(3)$-actions (but this will not be published).

3.Smooth non-compact Lie group action. In (1), I constructed infinitely many smooth $SL(m, \mathbb{H}) \times SL(n, \mathbb{H})$-actions on $S^{4(m+n)-1}$ by using smooth $\mathbb{R}^2$-actions on S^7 induced from vector fields.

In (2), I gave the another proof of the Kuiper's theorem $\mathbb{C}P(2)/\mathrm{conj} \cong S^4$ from the transformation group theoretical point of view, and by using this fact I could construct a continuous $SL(3, \mathbb{R})$-action on S^4 whose restricted $SO(3)$-action is the conjugated action.