Results

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My research area is Cartan geometry. Results which we obtained until now are as follows.

1. Equivalence problem of second order PDE.

First, we introduce a notion of the equivalence problem of differential equations in general. We need to fix classes of differential equations and a group of coordinate transformations to consider this problem. Then, the equivalence problem of differential equations is a problem how differential equations change under coordinate transformations. We can also express this problem in terms of group actions. Let $\mathcal{G}$ be a coordinate transformation group and X be a set of certain differential equations. Then the equivalence problem for differential equations in X is interpreted as the problem of determining the orbit decomposition with respect to the action of $\mathcal{G}$ on X. For this problem, we studied an equivalence problem of second order PDE with respect to scale transformations. Consequently, we calculated local invariant functions of this problem by using Cartan's method. Moreover, we also considered symmetry of the flat equation which have maximal symmetry corresponding to transformation groups $\mathcal{G}$ of equivalence problems.

2. Degenerations of implicit second order PDEs.

This thema is based on a joint work with Kazuhiro Shibuya. We studied some degenerations of implicit second order PDEs for one unknown function of two variables. Classically, it is well-known that Monge, Darboux, Goursat, Cartan, and the others studied geometric structures of implicit PDEs. However, there are uncharted territories for geometric studies of second order PDE. In particular, many of well-known results are obtained under a certain regularity condition. Hence, it is natural to consider structures of implicit PDEs which do not satisfy this condition. To consider the problem by using differential systems, we assumed the appropriate condition. Under the condition, we obtained some characterization of these equations.

3. An explicit construction of special Lagrangian fibration.

We also considered an explicit method of constructing special Lagrangian submanifolds as an application of Cartan geometry to another research area. In particular, we constructed special Lagrangian fibration in a certain Hyper-Kähler 2-fold which is called Taub-NUT space by using techniques of group symmetry (or associated moment map) and moving frame.