

Plan of Research

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In various occasions, it has been noticed that certain aspects of four dimensional quantum field theories resemble those of two dimensional QFTs. In particular, for theories with conformal symmetries, such as string theories and gauge theories. This has close connection with integrability.

Recently, a large class of $\mathcal{N} = 2$ superconformal $SU(n)$ quiver gauge theories in four dimensions are constructed by Gaiotto. Subsequently an interesting conjecture has been made by Alday, Gaiotto and Tachikawa (AGT). The AGT conjecture propose the equivalence of the Nekrasov partition function of the $SU(2)$ quiver gauge theory and the 2d conformal block of the Liouville theory. This conjecture is soon generalized to $SU(n)$ cases. It states that the Nekrasov partition functions of the $SU(n)$ quiver gauge theories can be expressed by using the conformal blocks of the 2d conformal Toda field theories. These are followed by a number of extensive checks and pieces of supporting evidence.

The Nekrasov partition function is a regularization of the instanton partition function of the gauge theory on the so-called Ω -background. It contains regularization parameters ϵ_1 and ϵ_2 . At $\epsilon_1 = -\epsilon_2 = 0$, the leading part of the (logarithm of) partition function reproduces the Seiberg-Witten prepotential.

Very recently, Dijkgraav and Vafa have suggested an explanation of the AGT conjecture by using the so-called quiver matrix model. By choosing “multi-Penner” (i.e., “multi-log”) potentials for the A_{n-1} quiver matrix models, they argued that the spectral curve of the matrix model at large N (the size of the matrix) can be understood as the Seiberg-Witten curve of the corresponding $SU(n)$ generalized quiver superconformal gauge theory.

Inspired by these new developments on 2d-4d conformal connection, Itoyama, Maruyoshi and Oota studied β -deformed quiver matrix models. For the case of $SU(n)$, we proposed a quantum spectral curve at finite N . For $n = 3$, residue analysis is provided both for the curve of the matrix model and for the Seiberg-Witten curve of the $SU(3)$ quiver gauge theory with six flavours, leading the the matching of the mass parameters.

We plan to continue this study on the properties of the β -deformed quiver matrix model and its quantum spectral curve. The deformation parameter β of the matrix model is related to the Nekrasov’s deformation parameters ϵ_1 and ϵ_2 . It will be interesting if the equivalence of the Nekrasov partition function and correlation functions of the deformed matrix model be shown beyond the leading part (or the Seiberg-Witten curve). The β -deformed matrix model has the 2d CFT representation in terms of chiral bosons, the equivalence will provides a “proof” of the AGT conjecture. And we expect that these will give us more deep understanding on the 2d-4d conformal connection.