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Results

For a closed symplectic manifold, we can construct the moduli space of stable maps. Gromov-Witten invariant is a symplectic invariant defined as an integration over the moduli space. Gromov-Witten invariants produce the (small) quantum cohomology. Gromov-Witten theory is an important theory, but it is very difficult to calculate Gromov-Witten invariants by definition because the moduli space is complicated.

Gromov-Witten invariants also produce the quantum differential equation (QDE), which is a system of PDE. We need to calculate Gromov-Witten invariants before we give the explicit description QDE by definition. But we can obtain the description without computing explicit value of the Gromov-Witten invariants in the case of symplectic toric manifolds and their complete intersections. Therefore we expect the theory of recovering Gromov-Witten invariants from the QDE, but the explicit description of QDE never directly gives the Gromov-Witten invariants. To solve this problem, Guest conjectures that the Birkhoff factorization of the transpose of the fundamental solution of the QDE recovers Gromov-Witten invariants.

We prove the Guest's conjecture for Fano hypersurfaces of a complex projective space in [4]. Moreover we give an algebraic algorithm for computing Gromov-Witten invariants. We define "adapted family of flat connections", "a pair of D-module and adapted basis" and "adapted differential equation". We give relation among them. Using the relations we prove that a uniqueness of a representative of adapted gauge equivalence class of adapted family of flat connections. We also gives a necessary and sufficient condition for an adapted gauge equivalence. These results give the proof of Guest's conjecture for Fano hypersurfaces of a complex projective space.

We study the QDE of a weighted projective space in the joint work with M. Guest [3]. If we consider quantum cohomology of a manifold, the total cohomology is often assumed to be generated by the second cohomology. This assumption plays a very important role. But such an assumption never holds for orbifolds. This fact makes our situation complicated. But in the case of weighted projective spaces, we conclude that the pair of an "abstract quantum cohomology" obtained by a "factorization" of QDE and a bilinear form obtained by considering the dual of quantum D-module is identified with the pair of the quantum orbifold cohomology and the orbifold Pincaré intersection form.

We construct a grading structure on a D-module which is modelled on quantum D-modules and we extends the above theory for Fano hypersurfaces [2]. This prove that Guest's conjecture for wider class containing monotone symplectic toric manifolds.

Starting from the quantum differential equation associated to a weighted projective space, we construct a Frobenius manifold [1]. We see that the Frobenius manifold coincides with the big quantum cohomology of the weighted projective space.