Research plan

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In the adjoint representations of a connected reductive algebraic group G, the main theorem of the Kempf-Rousseau theory asserts that the existence of optimal cocharacters for any nilpotent element. The optimal cocharacters have various characters. After the optimal cocharacters (that added condition a little) proved to be the associated cocharacters by Premet, some problems in good characteristic are solved by using this theory. In 2008, R. Fowler and G. Röhrle proved the Jantzen's expectation such that the cocharacters of H associated to some $X \in \mathrm{Lie}(H)$ are precisely the cocharacters of G associated to X that take values in H if H is the closed reductive subgroup of G and if G and H have same rank.

Actually, the Kempf-Rousseau theory is characteristic-free. Therefore, we can also use this theory for bad characteristics. However, the correspondence of nilpotent elements and the optimal cocharacters are a little abstract. If we can more concretely grip this correspondence by some methods, it is possible that we can find some new results. For example, if we prove that the finiteness of the number of orbits in the set of nilpotent elements in $\mathrm{Lie}(G)$ which have the same optimal cocharacter λ of G, then we can prove that the number of orbits in $\mathrm{Lie}(G)$ is finite.

From the above-mentioned background, we will reseach the Kempf-Rousseau theory itself, and the nilpotent orbits by using this theory and the latest theory.