Plan

I plan the research on generalized function-type potentials and boundary conditions in quantum mechanics. I especially aim at one-dimensional system whose Hamiltonian is given by second-order differential operator as follows:

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x).$$

It is known that the system with the generalized function-type interaction, for example delta-function potential, result in boundary problems at the support of the generalized function. Kurasov [1] considered 4-parameter family of generalized functions and showed that an element of the family corresponds to a boundary condition that is permitted physically (1). But the correspondence in [1] is not complete; there exist boundary conditions that do not correspond to any generalized functions proposed in [1]. Moreover, I find the class of generalized functions have extension characterized by 16-parameters(2). Since the boundary conditions permitted physically are characterized by 4 real numbers [2], there exists redundancy in the correspondence of the generalized function-type potentials to the boundary conditions. In order to clarify the situation, I plan to investigate the complete correspondence of a boundary condition to the class of generalized function-type potentials. I already find a class of generalized function-type potentials that corresponds to the smooth boundary condition, in other words, no generalized function-type potential case. The smooth boundary class of the generalized functions includes generalized functions with arbitrary size coupling constant. This fact implies that a strong coupled generalized function and a weak one give an identity boundary condition. It implies explicit strong-weak correspondence in quantum mechanics.

I consider that the strong-weak correspondence in quantum mechanics is important in following two meanings. Firstly, the strong-weak correspondence enables us to perturbative calculation in strong coupled generalized function-type potential. It is known that the path integral of systems with delta-function potentials can be done exactly by using the perturbation [3]. The perturbation method is easily applied to systems with generalized function-type potentials then the correspondence will be useful. Such pass integrals lead to partition functions; therefore exact thermodynamic property of the system with boundary conditions, which is interested recently and studied in [5,6], are obtained. As another application, I expect to obtain propagator of system in the quantum graph that is one-dimensional system with blanches. Secondly, through the study of field theories that result in quantum mechanics Hamiltonian with generalized function-type potentials studied in [7]etc., I expect to find strong-weak correspondence in field theory. For these two future views, I believe that this research on the generalized function and boundary conditions is important. In order to carry out this research, I would like to go to Czech Technical University and have a discussion with Prof. Pavel Exner who is the authority of this field.

Reference

[1] P. Kurasov, J. Math. Anal. Appl. **201** (1996) 297.

[2] I. Tsutsui, T. Fülöp, T. Cheon, J. Phys. A: Math. Gen. **36** (2003) 275.

[3] C. Grosche, Annalen Phys. 2 (1993) 557.

[4] T. Cheon, P. Exner, O. Turek, Annals of Physics (2010) 325.

[5] T. T. Fülöp, H. Miyazaki, I. Tsutsui, Mod. Phys. Lett. **A 40** (2003) 2863.

[6] N. Yonezawa, Prog. Theor. Phys. **123** (2010) 35.

[7] A. Kundu, Phys.Rev.Lett. 83 (1999) 1275.

---

(1) It is known that some boundary conditions break self-adjointness of Hamiltonian; therefore, a class of boundary conditions that maintain the self-adjointness of Hamiktonian is important in the context of physics.

(2) This extension is essential in order to discuss the quantum graph.