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Plans of my research (Nobutaka Boumuki)

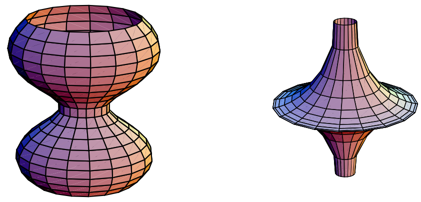
A partial differential equation $Au_{xx}+Bu_{xy}+Cu_{yy}+(\text{lower order terms})=0$ is called *elliptic* (resp. *hyperbolic*), if $B^2-4AC<0$ (resp. $B^2-4AC>0$). Here u, A, B and C are real functions with respect to the variables x and y.

I'm interested in relation between elliptic PDE and hyperbolic PDE; in particular, relation between the sinh-Gordon equation and the sine-Gordon equation:

$$u_{z\bar{z}}+\sinh u=0 \quad\longleftrightarrow\quad u_{xy}-\sin u=0, \tag{1}$$

where the word "PDE" means partial differential equations. On the one hand, surfaces of constant mean curvature (or CMC-surfaces, for short) in $\mathbb{R}^3$ are closely related with the sinh-Gordon equation. On the other hand, surfaces of constant negative Gaussian curvature (or K-surfaces, for short) in $\mathbb{R}^3$ are closely related with the sine-Gordon equation. These imply that the study of relation between CMC-surfaces and K-surfaces is closely related with the study of the relation (1).

$$\text{CMC-surfaces in } \mathbb{R}^3 \quad\longleftrightarrow\quad \text{K-surfaces in } \mathbb{R}^3. \tag{2}$$



It is well-known that for a CMC-surface M in $\mathbb{R}^3$, its Gauss map $\nu : M \to S^2$ is harmonic with respect to the induced metric, and that for a K-surface $\bar{M}$ in $\mathbb{R}^3$, its Gauss map $\bar{\nu} : \bar{M} \to S^2$ is Lorentz harmonic with respect to the second fundamental form. The loop group method enables us to interrelate ν with $\bar{\nu}$, and therefore it might enable us to clarify the relation (2). Then we might clarify the relation (1). For the reason we guess that the loop group method will enable us to clarify the relation (1).

My aim is to clarify the relation (1) by means of the loop group method.

That's all