Summary research

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Geometric analysis on Alexandrov spaces ([1]). In the attempt to generalize the Ricci lower bound condition on metric measure spaces that do not have a differentiable structure in the classical sense, many mathematicians have introduced new definitions of the Ricci lower bound condition, for example the curvature-dimension condition, introduced by Sturm, Lott and Villani, the measure contraction property, introduced by Ohta, etc. The weakest condition introduced for Alexandrov spaces is the so-called infinitesimal Bishop-Gromov condition introduced by Kuwae and Shioya. In [1] we derive the parabolic Harnack inequality for such spaces and we prove some Liouville type theorems and heat kernel estimates.

Weak convergence of laws of stochastic processes on non-compact manifolds ([2]). In [5] Ogura studied the weak convergence of the laws of the Brownian motions on compact Riemannian manifolds. More precisely, he proved that, if a sequence of compact manifolds $(M_n)_{n\in\mathbb{N}}$ converges to a compact metric space with respect to the measured Gromov-Hausdorff convergence and the Kasue-Kumura's spectral distance, then, the laws of the processes on M_n converge to the law of the Markov process associated to the transition density function of the limit space. The convergence with respect to the Kasue-Kumura's spectral distance may be ensured, for example, by a lower bound condition of the Ricci curvature. In [2] we study the same problem dropping the assumption of compactness of the manifolds using a generalization of the Mosco convergence theorem proved by Kuwae and Shioya.

The radial process on Alexandrov spaces ([3]). We try to extend to Alexandrov spaces with curvature bounded below a result which is well-known for Riemannian manifolds, namely that the radial process is a semimartingale. The proof of this result needs an upper bound for the drift of the radial process. For Riemannian manifolds the existence of this upper bound is ensured by the fact that on every geodesic ball the sectional curvature is bounded from above. We are looking for some as weak as possible conditions to obtain the same result for Alexandrov spaces for which there is no notion of curvature bounded by above.

L^p-independence of spectral bounds of Feynman-Kac semigroups ([4], joint work with Kim and Kuwae). We investigate the L^p-independence of the spectral bound of Feynman-Kac semigroup by continuous additive functionals which do not necessarily have bounded variation and whose Revuz measures are smooth measures of Kato class in the framework of symmetric doubly Feller processes. Examples of Cauchy principal value and Hilbert transform of Brownian local time, and for relativistic symmetric stable processes are presented. Our theorems extend preceding results obtained by Simon, Sturm, Takeda, and Zhang.

References

[1] G. De Leva, *Parabolic Harnack inequality on metric spaces with a generalized volume property*, to appear in Tohoku Math. Journal.

[2] G. De Leva, *Weak convergence of laws of stochastic processes on non-compact manifolds*, submitted.

[3] G. De Leva, *The radial process on Alexandrov spaces as semimartingale*, in preparation.

[4] G. De Leva, D. Kim, and K. Kuwae *L^p-independence of spectral bounds of Feynman-Kac semigroups by continuous additive functionals*, J. Funct. Anal. **259** (2010), no. 3, 690-730.

[5] Y. Ogura, *Weak convergence of laws of stochastic processes on Riemannian manifolds*, Probab. Theory Relat. Fields 119 (2001), 529–557.