Research program

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- **Symplectic structures on small covers and real moment-angle manifolds**
  A small cover of dimension n is a closed manifold on which $(\mathbb{Z}_2)^n$ acts effectively and whose orbit space is a simple convex polytope. Among small covers, there are many aspherical manifolds. As well as toric manifolds, each small cover corresponds to a combinatorial object which is a so-called characteristic pair. A necessary and sufficient condition is known for a small cover to be orientable. The author gave a necessary and sufficient condition for a real Bott manifold, which is a small cover over a cube, to have a symplectic structure. But, we do not know for other small covers to have a symplectic structure. The purpose of this study is to characterize all small covers to have a symplectic structure. It is very interesting even the case when the orbit space is a product of some polygones. The author also wants to work on real moment angle manifolds, which are finite covering of small covers.
- **Manifold on which a torus acts and whose fixed points are isolated**
  To a torus manifold , we can assign a multi-fan introduced by A. Hattori and M. Masuda. The multi-fan associated with a torus manifold allows us to compute some invariants. To a GKM space, we can assign a GKM graph which is a labeled graph. The GKM graph allows us to compute its cohomology ring. Similarly to torus manifolds and GKM spaces, the author wants to assign a combinatorial object to a manifold on which a torus acts under a special condition in order to study torus actions. Especially,
  1. symplectic action on a symplectic manifold with an isolated fixed point,
  2. topology of complex torus manifolds.

  With respect to 1, it is known that a circle action on a Kähler manifold has a fixed point if and only if the action is Hamiltonian. But, we do not know whether the action is Hamiltonian or not for a symplectic action except some spacial cases.
  With respect to 2, M. Masuda and the author showed that a complex manifold whose odd-degree cohomology groups vanish has the same cohomology ring as a toric manifold. But, we do not know any example of nontoric complex torus manifolds.