Plan of my research for the future

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The purpose of my research is to study the following 3 problems.

A knot contained in a spatial graph:
We consider a spatial embedding f of the complete graph K_n with n vertices. We denote the maximum number of crossing numbers of knots contained in $f(K_n)$ by $c(f(K_n))$, and the minimum number of $c(f(K_n))$ in every spatial embedding by $\alpha(n)$. Conway-Gordon proved that every spatial embedding of the complete graph with 7 vertices contain a nontrivial knot. In other word, $\alpha(7) = 3$. We are trying to evaluate the values of $\alpha(n)$. First problem is that $\alpha(2n-1) = \alpha(2n) = 2n-5$? for an integer $n \geq 4$. Since we proved that "every column embedding of the complete graph with $(2n-1)$ vertices or $2n$ vertices contains the torus knot of type $(2n-5, 2)$", we conjectured the following equality.

We are trying to solve this problem by the following a method. The method is to deal with any crossing changes of column embeddings. We are studying in the case of $n = 5$. We proved "for a column embedding of the complete graph with 9 vertices, the spatial graph dealt with one crossing change at a vertex that appears on the xy-plane contains a knot whose crossing number is greater than or equal to 5". Conway-Gordon proved that every spatial embedding of the complete graph with 7 vertices contain a nontrivial knot by using the Arf invariant. Note that we can not evaluate the crossing number of a knot by the Arf invariant. So, in order to evaluate crossing numbers we try to construct an invariant which is not changed by any crossing change.

The knot table by circular numbers:
We are try to classify knots by column embeddings. For a knot K, we proved the following relations between the circular number $\mathrm{Circ}(K)$ and the crossing number $c(K)$; (1) $\mathrm{Circ}(K) \leq c(K) + 2$, (2) if $\mathrm{Circ}(K) \geq 2$, then $2\{\mathrm{Circ}(K)\}^2 - 3\mathrm{Circ}(K) \geq c(K)$. As a result, it may be meaningful to make the table of knots by circular numbers. In the case of $\mathrm{Circ}(K) = 3$, we proved that K is a trefoil knot. We consider the case of $\mathrm{Circ}(K) \geq 4$. If a knot K is a prime knot with circular number 4, then is K the 8-figure knot?

Since the projections of a circle to a plane are ellipses, the combinatorial number of immersions of 4 ellipses is finite. The number of projections involved in 4 ellipses is also finite. Thus, the number of knots associated with circular embedding and such a projection is finite. We believe that we can solve this problem in this way. Since lengths of major and minor axes of ellipses are not fixed, this problem is too complicated. We might approach to the property of circular numbers by making an algorism.

The relation of the number of digraphs and ordinary graphs:
For even number n, Read proved that the number of self-complementary ordinary graphs with $2n$ vertices is equal to the number of self-complementary digraphs with n vertices. We consider the similar problem for edge colored graphs. We define cyclic automorphism as a generalization of self-complementarity, and we obtain formulas of the number of cyclic automorphic digraphs and ordinary graphs. By using these formulas, we consider a generalization of Read's formula, that is a relation between the number of cyclic automorphic digraphs and the number of cyclic automorphic ordinary graphs.

We are trying to solve this problem by the following two methods. One is that consider Read's formula as the formula for 2-colored graphs and notice the common divisor of edge numbers of corresponding graphs. It is not easy to compare formulas that we obtained for the number of cyclic automorphic digraphs and ordinary graphs, because of the difference of their isomorphisms. However, we will try to construct their correspondence inductively. Another method is that construct cyclic automorphic ordinary graphs by using cyclic automorphic digraphs. We can compute the number of cyclic automorphic graphs by using our results, but they are not suitable for constructing graphs. Although it is difficult to construct all cyclic automorphic graphs explicitly, we have experience and technique on it.