Results

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My research thema is geometric study of partial differential equations. Results which we obtained until now are as follows.

1. Equivalence problem of second order PDE.

First, we introduce a notion of the equivalence problem of differential equations in general. We need to fix classes of differential equations and a group of coordinate transformations to consider this problem. Then, the equivalence problem of differential equations is a problem how differential equations change under coordinate transformations. For this problem, we studied an equivalence problem of second order PDE with respect to scale transformations. Consequently, we calculated local invariant functions of this problem by using Cartan's method.

2. Degenerations of second order single PDEs.

This thema is based on a joint work with Kazuhiro Shibuya. We studied some degenerations of implicit second order PDEs for one unknown function of two variables. Classically, it is well-known that Monge, Darboux, Goursat, Cartan, and the others studied geometric structures of implicit PDEs. However, many of well-known results are obtained under a certain regularity condition. Hence, it is natural to consider structures of implicit PDEs which do not satisfy this condition. To consider the problem by using differential systems, we assumed the appropriate condition. Under the condition, we obtained some characterization of these equations.

3. Geometric study of type-changing equations.

This thema is based on a joint work with Kazuhiro Shibuya. Type-changing equations $\Sigma = \{F = 0\}$ are introduced as follows. For second order regular PDEs, we can consider the discriminant Δ. On a smooth hypersurface $\Sigma = \{F = 0\}$ defined by F, a point v is called hyperbolic, elliptic, or parabolic when the sign of $\Delta(v)$ is negative, positive, or zero, respectively. If we take a hyperbolic point or an elliptic point as a base point, points around the base point are also hyperbolic or elliptic respectively. However, this property is not satisfied for parabolic points. That is, if a hypersurface $\Sigma = \{F = 0\}$ have a parabolic point, there is a possibility that Σ is not locally parabolic. We call these PDEs type-changing equations. We clarified fundamental properties of these equations and formulated the notion of solutions of second order regular PDEs. Moreover, for a special type of solutions which are called parabolic solutions, we gave an exsistence condition of these solutions.