

My research plan consists of three parts as follows;

(1) Lorentzian Ricci solitons

In this research, we construct Ricci solitons on pseudo-Riemannian manifolds. In particular, we study Sol-soliton. And we will write a paper about Lorentzian Ricci solitons on the higher-dimensional classical Heisenberg group.

(2) Properties of Ricci solitons on Lorentzian manifolds

The paper [1] prove that the group of rigid motions of Euclidean 2-space has Lorentzian Ricci solitons. The metrics are not Einstein. W. Batat, M. Brozos-Vazquez, E. Garcia-Rio, S. Gavino-Fernandez(2010) prove that Egorov spaces and ε -spaces have Lorentzian Ricci solitons and there are non-rigid. This leads to new examples of Lorentzian Ricci solitons without Riemannian analog. I would like to clarify properties of Ricci solitons in Lorentzian geometry.

(3) Study of on cohomogeneity one metrics

There exists a left-invariant coframe $\{\theta^i\}_{i=1}^3$ on three-dimensional unimodular Lie group G satisfying $d\theta^i = 2\theta^j \wedge \theta^k$, where (i, j, k) are cyclic permutation of $\{1, 2, 3\}$. Then cohomogeneity one metrics with respect to G is described as

$$g = dt^2 + a(t)^2(\theta^1)^2 + b(t)^2(\theta^2)^2 + c(t)^2(\theta^3)^2. \quad (1)$$

For which $\{a(t), b(t), c(t)\}$ the resulting cohomogeneity one metrics are several metrics, for example, constant curvature metrics and product metrics. I will study for which $\{a(t), b(t), c(t)\}$ the resulting cohomogeneity one metric is Einstein or Ricci soliton. We have already proved that there are cohomogeneity one Ricci-flat metrics with respect to $E(2)$. We will consider that $\{a(t), b(t), c(t)\}$ has the kind of orbit.