We explicitly calculate the Riemannian curvature of D-dimensional metrics recently discussed by Chen, Lü and Pope. We find that they can be concisely written by using a single function. It is shown that the metrics are of type D ([19] of Publication List).

Assuming the existence of a single rank-2 closed conformal Killing-Yano tensor with a certain symmetry we show that there exist mutually commuting rank-2 Killing tensors and Killing vectors. We also discuss the condition of separation of variables for the geodesic Hamilton-Jacobi equations ([20]).

We classify all spacetimes with a closed rank-2 conformal Killing-Yano (CKY) tensor. It is shown that the *D*-dimensional Kerr-NUT-de Sitter spacetime constructed by Chen-Lü-Pope is the only spacetime admitting a non-degenerate closed CKY tensor ([21]). For spacetimes with general closed CKY tensor, they give a generalization of Kerr-NUT-de Sitter (KNdS) spacetimes. The metric of the generalized KNdS spacetime is a metric of the fiber space whose base spaces are several Kähler spaces and at most one general space, and its fiber is the KNdS space ([24, 25]).

It is shown that the Dirac equations in general higher dimensional Kerr-NUT-de Sitter spacetimes are separated into ordinary differential equations ([22]).

In the generalized Kerr-NUT-de Sitter spacetime, it is shown that certain type of tensor perturbation admits separation of variables and the linearlized Einstein equation (the Lichnerowicz equation) turns into ordinary differential equations ([26]).

We consider β -deformation of the quiver matrix models in the light of the recent progress on 2d-4d connection of conformal field theories, in particular, the Alday-Gaiotto-Tachikawa (AGT) conjecture. A quantum spectral curve is introduced in the case of SU(n) quiver matrix model. Residue analysis is provided both for the curve of the SU(n) matrix model with the "multi-log" potential and for the Seiberg-Witten curve in the case of SU(n) with $N_f = 2n$ flavors, leading to the matching of the mass parameters. The isomorphism of two curves is made manifest ([27]).

We observe that the Dotsenko-Fateev integral representation of the conformal block of 2d conformal field theory can be interpreted as a β -deformed matrix model of Selberg type. Using the formula associated with the Jack polynomials, we established the method of generating q-expansion coefficients for conformal block and Nekrasov function for $\mathcal{N}=2$ supersymmetric SU(2) gauge theory with four flavours ([28]).

We then consider a series of massive scaling lmits of the β -deformed matrix model of Selberg type (SU(2)) with $N_f = 4$ which reduce the number of flavours to $N_f = 3$ and subsequently to $N_f = 2$ ([29]).