The Plan of Research

I will mainly continue the study of CR geometry, and for my future study, I will try to find a new subject to research.

§3. The study of CR geometry

As I mentioned in the Overview, my present main problems are the CR Q-curvature flow and the pseudo-Einstein problem. To investigate the problem of the CR Q-curvature flow, first, I will study the essential positivity of the CR Paneitz operator. Related to this problem, we can consider some kinds of the interesting generalization of this problem. One important generalization of this problem is to find a contact metric with its Q-curvature equals a prescribed function. This kind of study is well developed by S.Brendle in Riemannian manifold case, and it is known that the blow-up phenomena occur if the prescribed function is not constant.

To consider the pseudo-Einstein problem, I would like to investigate the geometrical meaning of the condition $[\Delta_b, \overline{\partial}_b^*] = 0$. The quantity $[\Delta_b, \overline{\partial}_b^*]$ vanishes if pseudo-Hermitian torsion $A_{\alpha\beta}$ vanishes. However, the examples of s.p.c. manifolds with $[\Delta_b, \overline{\partial}_b^*] = 0,\ A_{\alpha\beta} \neq 0$ are not known yet.

The first part of my research plan is summarized as follows:

(i) find a sufficient condition for the CR Paneitz operator to be essentially positive, and relax the condition used in the Preprint [7].

(ii) consider the prescribed Q-curvature problem for an s.p.c. manifold and consider a generalization problem for higher dimensional manifolds.

(iii) investigate the condition $[\Delta_b, \overline{\partial}_b^*] = 0$, and solve the pseudo-Einstein problem.

(iv) study the geometry of pseudo-Einstein contact manifolds.

(v) re-consider the pseudo holomorphic mapping theory for s.p.c. manifolds from a new viewpoint.

§4. New research

An s.p.c. manifold which has a K-contact structure is called Sasakian manifold. One of a big advantage to study the Sasakian geometry is that we can use a transversely holomorphic coordinate on a Sasakian manifold. The geometry using transversely holomorphic coordinates is called the transverse geometry and is a main tool to study Sasakian geometry ([Futaki, Ono, Wang]). However, general s.p.c. manifolds do not admit a transversely holomorphic structure and the above method is not available. I am interested in what kind foliation structure do s.p.c. manifolds admit.

An s.p.c. manifold has a natural sub-Riemannian structure. From this reason, I think the s.p.c. geometry have a relation with theory or applied mathematics. I would like to study the meaning of strongly pseudo-convexity in applied mathematics.

For the effective study in this direction, I plan the following steps.

(i) Unit sphere bundles over a manifold are s.p.c. manifolds, but they are not Sasakian manifolds in general. I will investigate the Reeb orbits of unit sphere bundles, by using foliation geometry.

(ii) I also want to study the geometry of transversely symplectic foliation.

(iii) I will study and obtain basic notions and ideas in applied mathematics which is related to s.p.c. geometry.