Overview of My Study

§1. Background, Basic notions

I am interested in the geometry or the geometric analysis of strongly pseudo-convex manifolds (s.p.c. manifolds). An s.p.c. manifold is considered as a odd-dimensional analogue of complex/symplectic manifold, and defined as follows:

Definition 1: An oriented smooth manifold (M^{2n+1}, θ, J) is called s.p.c. manifold if the contact 1-form θ and $J \in \Gamma(\mathrm{End}(M; \ker\theta))$ satisfy following conditions,

(i) $J^2 = -1, \qquad [\Gamma(\ker\theta)^{1,0}, \Gamma(\ker\theta)^{1,0}] \subset \Gamma(\ker\theta)^{1,0}$,

(ii) the 2-symmetric tensor $L(\cdot,\cdot) = d\theta(\cdot, J\cdot)$ is positive definite on $\ker\theta$.

For the study of the geometry of s.p.c. manifolds, a complex/symplectic geometrical method is often used. We can consider a CR version of the Serre duality theorem (Publication 4, Publication 5), the Bochner formula ([Chang, Cheng, Chiu]), or the Sasaki-Ricci flow.

§2. The results

Since last year, I have studied CR Q-curvature and the pseudo-Einstein problem for s.p.c. manifolds with Professor Chang Shu-Cheng of National Taiwan Univ. The Q-curvature for Riemannian manifolds is a one of a generalization of the Gaussian curvature for surfaces. Although the CR Q-curvature is a CR analogue of the Q-curvature, the CR Paneitz operator, which dominates a transformation law of CR Q-curvature, is more complicated object than the Paneitz operator for a Riemannian manifold. For example, the kernel of the CR Paneitz operator has infinite dimension if the manifold is embeddable. In the paper [Chang, Cheng, Chiu], the CR Q-curvature flow was introduced for finding a contact form such that its Q-curvature vanishes.

I and Professor Chang Shu-Cheng showed that long time existence and convergence of a solution if the CR Paneitz operator P_0 is essentially positive and subelliptic. (Preprint [7]) Additionally, we showed that the conditions which are used in [Chang, Cheng, Chiu] implies the subellipticity of the CR Paneitz operator, and that the CR Paneitz operator is subelliptic if the manifold is embeddable.

The pseudo-Einstein problem is a problem of finding a contact form θ such that $\mathrm{Ric}(\theta) = \lambda d\theta$ on $\ker\theta$. By J.M.Lee, it is conjectured that for any s.p.c. manifold with $c_1(T_{1,0}) = 0$, we can find a pseudo-Einstein contact form, but this conjecture is still open. My research of the pseudo-Einstein problem is in the coauthored paper [2]. Using a method similar to a method developed by J.M.Lee, it is shown that if $[\Delta_b, \overline{\partial}_b^*] = 0$ and $A_{\alpha\beta}{}^{,\alpha} = 0$ then the s.p.c. manifold has a pseudo-Einstein contact form. Here, $\overline{\partial}_b^*$ is the formal-adjoint of $\overline{\partial}_b$ and $A_{\alpha\beta,\gamma}$ is a covariant derivative of the pseudo-Hermitian torsion.

Before last year, I studied the complex Monge-Ampère equation (Publication [6]) and J-holomorphic mapping for s.p.c. manifolds (Publication [3]). I the paper [6], I solved the Monge-Ampère equation under a certain condition by the classical continuity method. In Publication [3] and Preprint [8], I showed the removable singularity theorem and the regularity theorem of J-holomorphic mappings for s.p.c. manifolds. I expect that these theorems can be applied to establish a CR version of the Gromov-Witten theory.