## Research plan

I study properties of links by studying their diagrams and local moves on them. I will continue the study above, and I will also study projections of links. The concrete plan of my research is as follows:

## Warping degree labeling

By giving each edge the value of the warping degree, we can apply the warping degree labeling to an oriented knot diagram. Since the warping degree labeling tells us a lot of information about crossings, I expect that a knot invariant obtained from the warping degree labeling is useful to study properties of an alternating knot. After defining the knot invariant, I would like to show the relations to other knot invariants including the unknotting number and the dealternating number.

## Warping polynomial

I am considering the warping polynomial of an oriented knot diagram which is obtained from the warping degree labeling. I will look into how different a knot diagram is from an alternating diagram by using the warping polynomial. I will also apply the span of the warping polynomial to the study of a knot projection. Further, I would like to study an Eulerian and Hamiltonian four-valent graph with a condition by considering a knot projection a four-valent graph.

## Region unknotting number

I will study in more detail the region unknotting number of a knot. For example, I would like to understand the relation to other knot invariants including the genus and the unkotting number. I will explain a more detailed relation between the region unknotting number and the crossing number of a knot. Cheng and Gao gave an application of my result in [5] to the study of a link projection. I expect that the region crossing change can be applied to many other studies of a knot, a knot diagram, and a link projection as above. I would like to consider applications of a region crossing change and the region unknotting number, and study properties of minimal diagrams and projections of a knot.

## Warp-linking degree and the complete splitting number

In [2], I considered the warp-linking degree and the complete splitting number of a link diagram, and raised a question which is about a relation between them. I will try solving the question by considering the results in [3]. I would like to understand the complete splitting number in more detail in terms of statistics like in [4].

