Research summaries

The mathematical model on chemotactic aggregation of slime mold is described as a drift-diffusion system. For this system it is well-known that the the possibility of blow-up solution depends on the space dimensions. In particular it is suggested that both global existence and blow-up may occur according the size of a certain mass in two dimensional case.

I have investigated the asymptotic behavior of bounded solutions to the following parabolic system of chemotaxis in the whole space.

$$\begin{cases} \partial_t u = \Delta u - \nabla \cdot (u \nabla v), & x \in \mathbb{R}^n, t > 0, \\ \partial_t v = \Delta v - v + u, & x \in \mathbb{R}^n, t > 0, \\ u(x,0) = u_0(x), v(x,0) = v_0(x), & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

Specifically the following results was obtained:

(1) every bounded solution to (1) decays to zero as $t \to \infty$, and the solution behaves like the heat kernel ([1], [7]).

(2) In the higher dimensional case there exists a unique global solution to (1) such that satisfies special decay properties at space infinity for small initial data. Furthermore the asymptotic expansions of solutions have been given ([2]).

(3) if the l-th moment of u_0 is finite for some $l \geq 1$, then $u(t)$ admits moment estimates of l-th order. Moreover, as an application we can give the more precise asymptotic expansions of bounded solutions to (1) ([3], [6]).

(4) The convergence rates of bounded solutions to (1) can be improved by taking into account the center of mass of such solutions. In particular it is seen that the convergence rate is optimal in the one or two dimensional case ([4], [5]).

There are two points that should be remarkable in my research. First if n-th moment of u_0 is finite, then a correction term is contained in the asymptotic expansions for (1), and there also appears a difference between the expansions in odd and even dimensional cases (see [3], [6]). This phenomenon do not occur in the asymptotic expansion for Navier-Stokes equations, which is one of the pioneer works in this direction. Next we observe that there is a deep relation between the center of mass and asymptotic behavior of solution to (1). Therefore we suggest that there appears such a relation in other nonlinear partial differential equations.