Research Statement

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Stefan Bergman discovered a kernel function which is now called the Bergman kernel. In general, it is hard to obtain an explicit formula of the Bergman kernel of a given complex domain. Therefore it is fundamental and important to find a domain with explicit Bergman kernel. Actually, up to now many mathematicians work in that direction.

My current research interest is to develop a new method to find a domain with explicit Bergman kernel. I am also interested in the applications of explicit formulas for various problems on the Bergman kernel. Especially I am interested in the Lu Qi-Keng problem.

In my research, I consider the Hartogs domain. I proved that the Bergman kernel of the Hartogs domain is expressed explicitly by the polylogarithm functions if its base domain satisfies a certain condition. The following domain $D_{n,m}$ is a concrete example which satisfies the condition:

$$D_{n,m} = \{(z, \zeta) \in \mathbb{C}^n \times \mathbb{C}^m; ||\zeta||^2 < e^{-\mu||z||^2}\}, \quad (\mu > 0).$$

As an application of my formula, I investigated the Lu Qi-Keng problem of the domain $D_{n,m}$. Lu Qi-Keng conjectured that if Ω is simply connected, then K is zero-free on $\Omega \times \Omega$. It is already known that this conjecture is false in general. A domain in $\mathbb{C}^n$ is called a Lu Qi-Keng domain if its Bergman kernel function has no zeros. The Lu Qi-Keng problem asks whether or not a given domain is a Lu Qi-Keng domain.

I proved the following result for the domain $D_{n,m}$.

Theorem. *For any fixed $n \in \mathbb{N}$, there exists a unique number $m_0(n) \in \mathbb{N}$ such that $D_{n,m}$ is a Lu Qi-Keng domain if and only if $m \geq m_0(n)$.*

I also found the Cartan-Hartogs domain satisfies the condition. The Cartan-Hartogs domain is defined as follows:

$$D_N = \{(z, \zeta) \in \Omega \times \mathbb{C}^m; ||\zeta||^2 < N(z, z)^\mu\}, \quad (\mu > 0),$$

where Ω is a irreducible bounded symmetric domain and N its generic norm. An explicit formula of the Bergman kernel of this domain was known by W. Yin. However the polylogarithm does not appear in his formula. Thus my formula is another expression of the Bergman kernel of the Cartan-Hartogs domain.