Results of Research

Shunsuke Yamana

1. **How many Fourier coefficients determine a holomorphic modular form?**
   It is a fundamental problem in the theory of modular forms to know how many Fourier coefficients determine a holomorphic modular form. A special case of our result [1] can be stated as follows: Siegel modular forms of degree greater than one with respect to the full Siegel modular group are uniquely determined by primitive Fourier coefficients.
2. **A newform theory of modular forms of half-integral weight**
   The plus space is a canonical subspace of the space of modular forms of half-integral weight. Kohnen has established a theory of newforms for plus spaces of level $4M$, where M is an odd square-free integer. In the joint work [2] with Masaru Ueda, Kohnen's result was extended to the case of level $4M$, where M is a square-free (not necessarily odd) integer.
3. **Maass relations in higher degrees**
   The Saito-Kurokawa lifting is a lifting which associates to an elliptic cusp form a Siegel cusp form of degree two, and its image is characterized by certain linear relations among Fourier coefficients, the so-called Maass relations. A generalization of the Saito-Kurokawa lifting was constructed by Tamotsu Ikeda and is now called the Ikeda lifting. Kohnen defined a generalization of the Maass relations and proved jointly with Kojima that it characterizes the image of the Ikeda lifting under a certain assumption. We obtain the characterization of the image of the Ikeda lifting by removing their assumption ([3]).
4. **The Ikeda lifting on quaternion half-spaces**
   Siegel studied holomorphic modular forms on the Siegel upper half-space, and Braun studied holomorphic modular forms on the Hermitian upper half-space. Holomorphic modular forms on the quaternion upper half-space was studied by Krieg. We construct an analogue of the Ikeda lifting on the quaternion half-space ([4]).
5. **Jacobi forms of degree one with matrix index**
   We show that a certain subspace of space of elliptic cusp forms is isomorphic as a Hecke module to a certain subspace of space of Jacobi cusp forms of degree one with matrix index ([5]). This is a partial generalization of the work of Skoruppa and Zagier.
6. **Degenerate principal series representations for $Sp(n,n)$ and $O^*(4n)$**
   A degenerate principal series for classical groups is a representation obtained by inducing a one-dimensional representation from the Siegel parabolic subgroup. We give a complete description of all points of reducibility and the composition series of the degenerate principal series representations for quaternionic unitary groups ([6]).
7. **A new extension of the Siegel-Weil formula**
   The Siegel-Weil formula identifies a value of an Eisenstein series with a theta integral. For symplectic, unitary and orthogonal groups, we prove that if the point at which the Eisenstein series is evaluated lies within the left half-plane determined by the functional equation, then the Eisenstein series attached to the standard sections coming from the Weil representation are holomorphic at that point and the Siegel-Weil formula is valid with no restrictions ([7]).