Research plan

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Ricci curvature on metric spaces. In the attempt to give a definition of Ricci curvature for metric spaces more general than Riemannian manifolds, the first important step was done in [2] by Bacri and Émery which define the Ricci curvature of a diffusion process on a Riemannian manifold and proved that the Ricci curvature of the ordinary Brownian motion is the usual one. This approach was generalized in the convergent works of Sturm, Lott and Villani ([7] and [4]), who define the so-called curvature dimension condition, and by Ohta in [6] who defined the so-called measure contraction property. Moreover, Ollivier, following a different approach, gave a definition of Ricci curvature in terms of transportation distance for Markov chains ([5]). We are interested in the case of Alexandrov spaces with curvature (i.e., sectional curvature in the sense of Alexandrov) locally bounded below. For such spaces there is one more definition: the so-called infinitesimal Bishop-Gromov condition (Kuwae and Shioya, [3]), which gives a Bishop-Gromov type volume inequality. We would like to investigate about the equivalence of the above definitions for such spaces. As far, it is known that the infinitesimal Bishop-Gromov condition is equivalent to the measure contraction property and weaker than the curvature dimension condition. Nothing is known about the relation between the definition given by Kuwae and Shioya and that of Ollivier. We would like to prove that the infinitesimal Bishop-Gromov condition is equivalent to that of curvature dimension and find some relations to the definition of Ollivier.

Diffusion Processes on tubular domains. Many physical phenomena take place in structures which have in some directions very small dimensions (for example the fluid motion in narrow tubes or the propagation of electromagnetic waves in wave guides). Such structures can be described from a mathematical point of view by means of metric measure spaces with some diffusion processes. These metric spaces tend in the Gromov-Hausdorff topology to a graph. The study of diffusion processes on graphs is much simpler and can be considered as first model to study the physical phenomenon. Hence, it is necessary to ensure that diffusion processes defined on tubular domains converge to the corresponding process obtained in the limit when the tubular domains shrink to a graph. This kind of problems has been extensively studied in the case of Neumann boundary condition and there is a rich literature on this subject. The case of Dirichlet boundary condition is more difficult and less well understood. Recently, in [1], an interesting result was obtained with Dirichlet boundary condition under the assumption that the diffusion processes are generated by a certain family of stochastic differential equations. We would like to study the same problem considering the case of diffusion processes associated to Dirichlet forms.

References

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